## PHYSICS

Paper: PHY 522

## (Nuclear Reactions and Nuclear Astrophysics)

Full Marks: 50
The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer any five questions.

1. (a) What is astrophysical S-factor?
(b) In a non-resonant capture reaction of a charged particle by a nucleus, occurring at a steller temperature of $T$, the average reaction rate per interacting particle pair, $\langle\sigma v\rangle v$ being the relative velocity between the two charged particles, can be written as

$$
\langle\sigma v\rangle \propto \int_{0}^{\infty} f(E) d E
$$

where function $f(E)=e^{-E / k t-\sqrt{E_{G} / E}}$ and $E_{G}=2 \mu c^{2}\left(\pi \alpha Z_{1} Z_{2}\right)^{2}, \mu$ being the reduced mass and $\alpha$ the fine structure constant.
(i) Show that $f(E)$ maximizes over an energy window with peak at

$$
E_{0}=(k T / 2)^{2 / 3} E_{G}^{1 / 3} .
$$

(ii) Assuming a Gaussian shape for over the energy window, show that the width of the window is given as

$$
\Delta=(4 / \sqrt{3})\left(E_{0} k T\right)^{1 / 2}
$$

(c) Consider the capture reaction ${ }^{12} \mathrm{C}(p, \gamma){ }^{13} \mathrm{~N}$ that proceeds through an isolated resonance in ${ }^{13} \mathrm{~N}$.
(i) Considering the ground state $\operatorname{spin}$ of ${ }^{12} \mathrm{C}$ as $0^{+}$, find out the spin of the resonance state for s -wave proton capture.
(ii) The energy of the resonance state is 421 keV . If the proton width is 31 keV and the gamma width is 0.5 eV , what will be the cross-section at the resonance energy? $\quad 2+(2+2)+(2+2)$
2. (a) What is the $3 \alpha$ process for ${ }^{12} \mathrm{C}$ formation in the He burning stage of steller evolution? What is the spin of the Hoyle resonance and why?
(b) The central iron core of a massive star grows due to deposition from outer shells. As its mass increases and approaches the Chandrasekhar Mass, two nuclear processes occur in the core. What are these processes and how do they generate the collapse of the core?
(c) Taking the total energy production rate in the core of the Sun, undergoing Hydrogen burning, as $5.1 \times 10^{7} \mathrm{MeV} . \mathrm{g}^{-1} . \mathrm{s}^{-1}$ and its luminosity as $2.4 \times 10^{39} \mathrm{MeV} . \mathrm{s}^{-1}$, find out
(i) the mass of Hydrogen used up in the process to maintain the Luminosity;
(ii) the number of processes converting $4 \mathrm{H} \rightarrow{ }^{4} \mathrm{He}$ required to generate the Luminosity;
(iii) the age of the Sun with the given rate of energy production. Given the mass of H as $1.6 \times 10^{-24} \mathrm{~g}$ and energy produced in converting $4 \mathrm{H} \rightarrow{ }^{4} \mathrm{He}$ is 26 MeV . $3+3+4$
3. (a) Show that the imaginary component of the complex optical potential must be negative to generate absorption of flux in a nuclear reaction.
(b) Taking the optical potentials for 40 MeV protons to be $V=50 \mathrm{MeV}$ and $W=10 \mathrm{MeV}$ and for 40 MeV alpha particles, $V=100 \mathrm{MeV}$ and $W=30 \mathrm{MeV}$, estimate the ratio of the absorption lengths of protons and alpha particles in nuclear matter.
(c) In a one neutron transfer reaction $A(d, p) B$, assuming the reaction takes place at the surface of the nucleus, establish the relation between the direction $\theta$ of the outgoing proton and the angular momentum transfer and show that for $l=0$ transfer the maximum occurs at $\theta=0$. $3+3+4$
4. (a) In heavy ion collision, under strong absorption approximation, show that the fusion cross-section at energy $E$ can be expressed as

$$
\sigma_{f u s}=\pi R_{B}^{2}\left(1-V_{B} / E\right)
$$

where $V_{B}, R_{B}$ are, respectively, the Coulomb barrier strength and the barrier radius.
(b) A $40 \mathrm{MeV}{ }^{4} \mathrm{He}$ particle fuses with a ${ }^{28} \mathrm{Si}$ target nucleus forming a ${ }^{32} \mathrm{~S}$ compound nucleus. Find the excitation energy of the compound nucleus. What should be the energy required for a ${ }^{16} \mathrm{O}$ projectile fusing with another ${ }^{16} \mathrm{O}$ target nucleus to produce compound nucleus ${ }^{32} \mathrm{~S}$ at the same excitation energy?
Given that the mass excesses for ${ }^{4} \mathrm{He},{ }^{16} \mathrm{O},{ }^{28} \mathrm{Si}$ and ${ }^{32} \mathrm{~S}$ are $2424.91 \mathrm{keV},-4737.08 \mathrm{keV}$. -21492.91 keV and 26015.50 keV , respectively.
(c) Show that a nucleus with mass $A$ and charge $Z$ is unstable against small deformation and undergoes fission if the ratio $Z^{2} / A$ is larger than a critical value of $\left(Z^{2} / A\right)_{c}=2 s_{a} / a_{c}$, where $a_{s}$ and $a_{c}$ are the coefficients of the surface and Coulomb contributions in the formula for binding energy of a nucleus as a function of $A$ and $Z$.
5. (a) A normal star in a state of hydrostatic equilibrium acts as a self-regulating system. Explain.
(b) In relativistic collision of two particles, the Lorentz invariant quantity $S$ is defined as $S=p_{1}+p_{2}$ where $p_{1}=\left(E_{1}, \vec{p}_{1}\right)$ and $p_{2}=\left(E_{2}, \vec{p}_{2}\right)$ are the four-momenta of particles 1 and 2 . Show that in a collision of symmetric particles, like proton on proton, $\sqrt{S}=2 E$ in the centre of mass and $E=E_{1}+E_{2}$ the total available energy. Note that $c=1$ units have been used in defining the relations.
(c) Give the typical values (order of magnitude will do) of (i) rise time of pulse from HPGe detector (ii) energy required to produce one electron-hole pair in HPGe detector (iii) reverse bias voltage for HPGe detectors (iv) temperature at which the HPGe detectors are operated. $3+3+4$
6. Write the truth table for implementing Compton suppression in a HPGe detector indicating the detector corresponding to each input. How do we quantify the quality of Compton suppression? What do the Xand the Y-axis of a spectrum, acquired with a radiation detector, represent?
Which property of a detector is indicated by the width of peaks in its spectrum? How are the widths different for spectra acquired with HPGe and NaI detectors? How will the position of the same peak differ in spectra acquired with 12- and 13-bit ADCs?

$$
2+2+2+2-2
$$

7. The strength of a ${ }^{57} \mathrm{Co}\left(\mathrm{T}_{1 / 2} \sim 272\right.$ days) source is $1_{\mu} \mathrm{Ci}$ on January 1,2021 . What is its evolved strength on January 1, 2022? If we add another sample, with activity $1{ }_{\mu} \mathrm{Ci}$, of the same source to the aforesaid sample on January 1, 2022, what is the total strength of the source on that date? Let us assume that the source decays by emission of $122 \mathrm{keV} \gamma$-ray only. It is placed in a detection setup of absolute efficiency $1 \%$ on January 1,2022 . What is the count accumulated for 122 keV in 1 hour? If we now place an $\mathrm{Al}\left(\rho=2.7 \mathrm{~g} / \mathrm{cm}^{3}, \mu=0.152 \mathrm{~cm}^{2} / \mathrm{g}\right.$ at 122 keV$)$ disk of thickness 5 cm between the source and the detector, what will be the accumulated counts of 122 keV in 1 hour? Will the accumulated counts increase or decrease, if we replace the Al absorber with a Pb one of same thickness? Consider the setup without the absorbers and suggest one change in the geometry that'll increase the efficiency of detection.
( $1 \mathrm{Ci}=3.7 \times 10^{10}$ disintegrations $/ \mathrm{s}$ )
