2021

MATHEMATICS — GENERAL

Paper : DSE-A-2

(Graph Theory)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Choose the correct alternatives : 1×10

(a) The number of vertices of a regular graph of degree 3 with 15 edges is

(i) 5 (ii) 10 (iii) 20 (iv) 45.

(b) Maximum number of edges in a simple connected plane graph of order n is

(i) $2n - 4$ (ii) $3n - 10$ (iii) $3n - 6$ (iv) $3n$.

(c) Number of vertices of a complete graph having 66 edges is

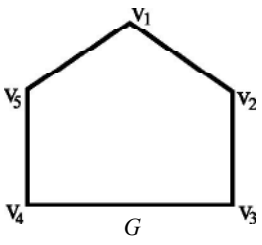
(i) 10 (ii) 11 (iii) 12 (iv) 13.

(d) The adjacency matrix of a graph G is always

(i) symmetric (ii) skew symmetric

(iii) singular (iv) non-singular.

(e)



G

G is

(i) bipartite and regular (ii) bipartite, but non-regular

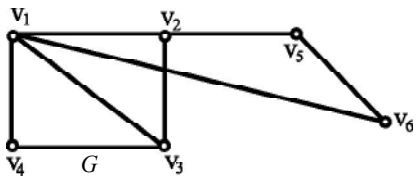
(iii) regular but non-bipartite (iv) neither regular nor bipartite.

(f) If $I(G)$ is an incidence matrix of a directed graph G without loops and non-directed edges, then each column of $I(G)$ contains

(i) two 1 (ii) one 1, one –1 (iii) two –1 (iv) one 1.

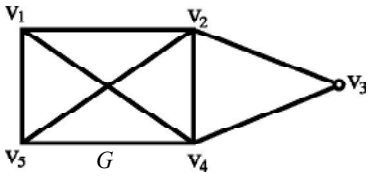
(g) The degree of the root of a binary tree is

(i) 0 (ii) 1 (iii) 2 (iv) 3.

(h) 

In the above graph G, distance between v_1 and v_6 is

(i) 0 (ii) 1 (iii) 3 (iv) ∞.

(i) 

G is

(i) non-planar and non-Eulerian (ii) planar and non-Eulerian

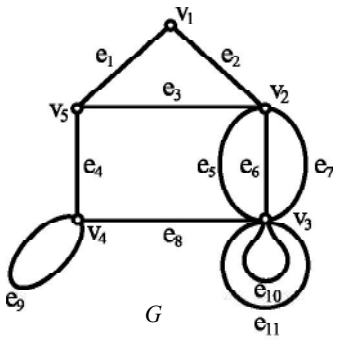
(iii) planar and Eulerian (iv) non-planar and Eulerian.

(j) The minimum number of pendant vertices in a tree with 5 vertices is

(i) 2 (ii) 3 (iii) 0 (iv) 1.

2. Answer ***any three*** questions :

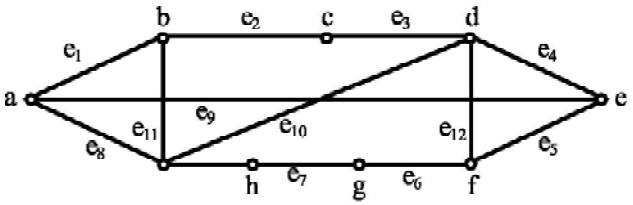
(a) (i) Define incidence matrix of a connected graph.

(ii) 

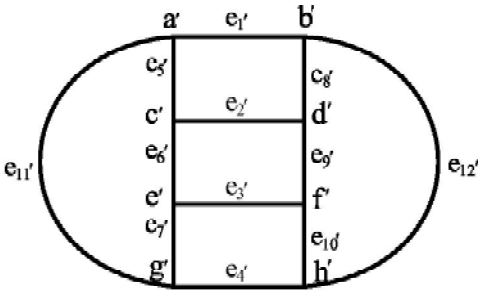
Find incidence matrix of G. 2+3

(b) (i) Define isomorphic graphs.

(ii)



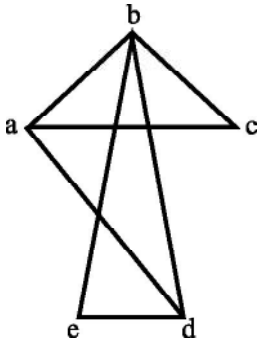
G_1



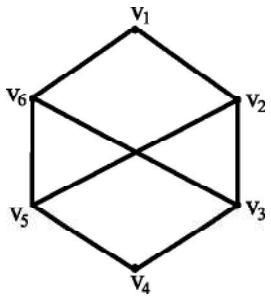
G_2

Is $G_1 \cong G_2$? Justify. 2+3

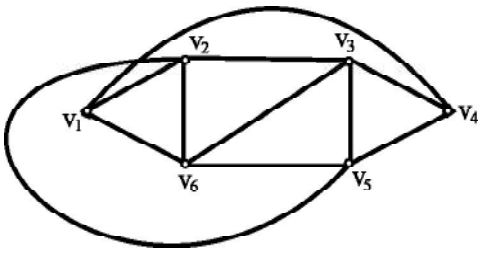
(c) Show that the following is a planar graph by redrawing it so that no edges cross. 5



(d) Define complement of a graph. Find the complement of the following graph. 2+3



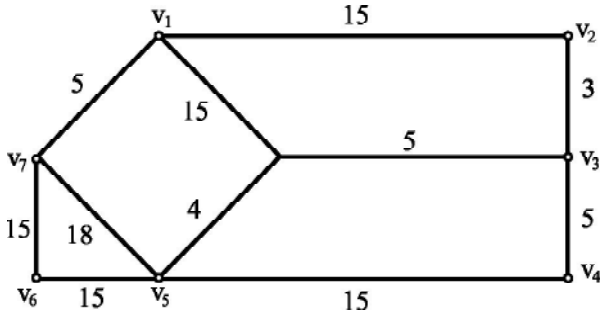
(e) What is a Hamiltonian graph? Is the following graph *G* Hamiltonian? Justify your answer. 1+1+3



G

3. Answer ***any four*** questions :

(a) (i) What is minimal spanning tree? Find minimal spanning tree of the graph given below :

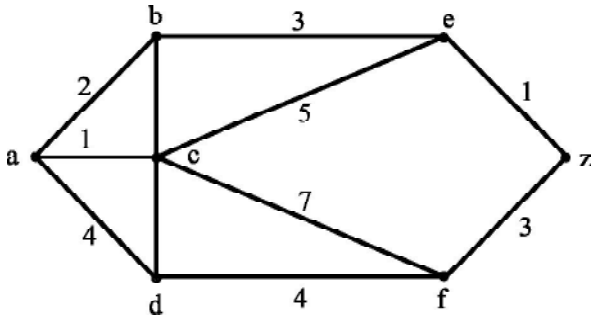


(ii) Prove that $K_{3,3}$ is non-planar. [(2+5)+3]

(b) (i) If degree of each vertex of a graph G is greater than or equal to 2, then show that G contains a cycle.

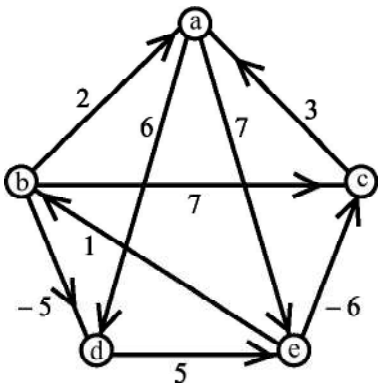
(ii) If G is a simple graph with at most $2n$ vertices and degree of each vertex is at least n, then show that G is connected. 5+5

(c) (i) Apply Dijkstra's algorithm to determine a shortest path between a to z in the following graph.



(ii) Draw a tree with 5 internal vertices and 5 terminal vertices. 7+3

(d) Using Floyd-Warshall algorithm, find the length of the shortest path between any pair of vertices a, b, c, d and e of the following weighted directed graph. 10



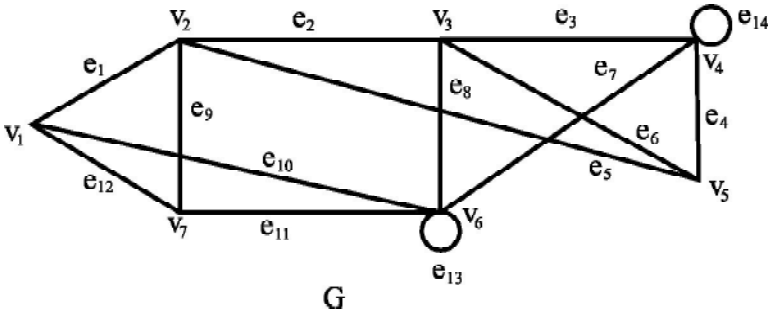
(e) (i) Draw a bipartite graph with degree sequence (1, 3, 4), (1, 2, 2, 3).

(ii) If G is a tree with all odd degree vertices, then show that number of vertices of G is even.

(iii) A tree has only vertices of degree 5 and degree 1. If the tree has 34 vertices, how many have degree 5? 5+2+3

(f) (i) Prove that a complete bipartite graph $K_{m,n}$ is Hamiltonian *iff* $m = n$.

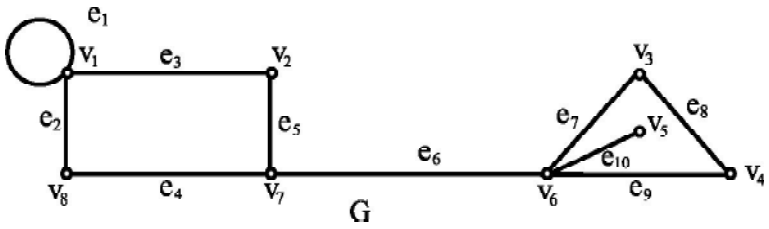
(ii)



G

Check if G is Eulerian and Hamiltonian or not. 5+5

(g) (i)



G

Find the faces and degree of each face in G. What is the relation between sum of degrees of faces and number of edges of G?

(ii) Does there exist a planar graph with 35 vertices and 100 edges?

(iii) Find the maximum number of vertices in a connected graph having 17 edges. 5+2+3