2022

PHYSICS

Paper : PHY 522

(Advanced Particle Physics)

Full Marks : 50

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Answer ***any five*** questions.

1. (a) Write down the matrix element $\mathcal{M}$ for muon decay in the low energy limit. Deduce $\overline{|\mathcal{M}|^2}$, expressing it as the product to two traces. (Do not evaluate the traces.)

   (b) What is the helicity of a neutrino in the Standard Model? Use the helicity argument to conclude that the rate of the decay $\pi_0 \to \nu\bar{\nu}$ should vanish if the neutrinos are massless.

   (c) The sum over polarizations $\sum_{r=1}^{3} \epsilon_\mu^r(k)\epsilon_\nu^{r*}(k)$ for a vector boson of mass M is of the form $Ag_{\mu\nu} + Bk_{\mu\nu}$. Find A and B, using the transverse condition and the normalization condition of the polarization vectors. (1+3)+(1+2)+3

2. (a) Write down the expression for the covariant derivative in the Standard Model. Obtain the mass of the W boson, given that $\langle\Phi\rangle = \begin{pmatrix} 0 & \frac{\upsilon}{\sqrt{2}} \end{pmatrix}^T$.

   (b) If Φ transforms as an SU(2) doublet, show that $\tilde{\Phi} = i\tau_2\Phi^*$ also transforms as an SU(2) doublet.

   (c) In the Standard Model, explain why $\left(\bar{u}_L \ \bar{d}_L\right)\Phi u_R$ is not $U(1)_Y$ invariant. (1+4)+3+2

3. (a) Draw all the tree-level diagrams contributing to $e^+e^- \to \mu^+\mu^-$. Why can the Higgs mediated diagram be neglected? Define the forward-backward asymmetry in terms of the differential scattering cross-section (no calculation is needed). Which diagram gives rise to the forward-backward asymmetry?

   (b) Show that $\Phi(x) = e^{i\zeta^a(x)\tau^a/\upsilon}\begin{pmatrix} 0 & \frac{\upsilon + H(x)}{\sqrt{2}} \end{pmatrix}^T$ fully parametrizes deviations from $\langle\Phi\rangle = \begin{pmatrix} 0 & \frac{\upsilon}{\sqrt{2}} \end{pmatrix}^T$ by expanding $\Phi(x)$ to first order in the fields.

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(c) Draw the lowest order diagrams for the decay of the Higgs into two gluons and into four charged leptons.

(d) Draw all the tree-level diagrams for $q\bar{q} \to GG$. (G denotes a gluon.) (1+1+1+1)+2+(1+1)+2

4. The formula for the differential scattering cross-section is :

$$d\sigma = \frac{1}{4\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \left(\prod_a \frac{d^3 p'_a}{(2\pi)^3 2E'_a} \right) \times (2\pi)^4 \delta^4 \left(p_1 + p_2 - \sum_a p'_a \right) |\mathcal{M}|^2$$

(a) Under what condition(s) can one write the initial state factor as $\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = E_1 E_2 |\vec{v}_1 - \vec{v}_2|$ in standard notations?

(b) Argue that the Dirac delta function appearing in the formula is Lorentz invariant.

(c) Suppose a process $a + b \to c + d$, where all four particles are Dirac fermions, is mediated through the following interactions in the Lagrangian :

$$\mathcal{L}_{\text{int}} = \bar{c}\gamma^\mu a X_\mu + \bar{d}\gamma^\mu b X_\mu,$$

where X_μ is a spin-1 particle. Draw the simplest diagram for the process, and write the expression for $\mathcal{M}$ assuming X to be very heavy compared to all other energies and masses. 3+2+(2+3)

5. Consider processes of the form $e + p \to \nu_e + X$, where X is arbitrary.

(a) Give an example of such a process if X consists of just one particle.

(b) If this case (i.e., only one particle in X) is considered in the rest frame of the proton, find the minimum kinetic energy of the electron that will be necessary for initiating the process. (No numberical work is necessary. Write everything in terms of masses of various particles. Assume the neutrino mass to be zero.)

(c) Give an example of such a process if X consists of two particles.

(d) Which electroweak gauge boson will be responsible for mediating such processes? 2+5+2+1

6. (a) Distinguish between the quark mass and flavour bases. How are the quark mass matrices in the mass basis related to those in the flavour basis? Derive an expression for the Cabibbo-Kobayashi-Maskawa matrix in terms of the above.

(b) For mesons involving the b-quark there are two different neutral meson systems. Identify them giving their quark structure. What is the advantage of the $\Upsilon(4S)$ in studying neutral B-mesons? What is the strategy behind an asymmetric B-factory? (1+2+3)+(1+1+2)

7. (a) For a two-neutrino system, derive the probability of neutrino oscillation to a different flavour and therefrom point out the essential ingredients for a non-zero probability.

(b) What are the interactions of neutrinos when they pass through matter and how do they affect neutrino mass? For a two neutrino system – ν_e and ν_μ – derive the impact of this effect on neutrino mixing and the mass-square difference. What is the relevance of this effect for solar neutrinos? 3+(2+4+1)