2021

MATHEMATICS — HONOURS

Paper : DSE-B-1

(Discrete Mathematics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer the following multiple choice questions (MCQ) in which only one option is correct. Choose the correct option with proper justification if any. 2×10

(a) Consider the lattice $S = \{1, 2, 3, 4, 6, 12\}$, the divisors of 12 ordered by divisibility. The complement of 3 is

(i) 1 (ii) 12

(iii) 2 (iv) 4.

(b) The number of unit element of the ring Z is

(i) 2 (ii) 1

(iii) 3 (iv) 0.

(c) The multiplicative inverse of 7 in Z_{11} is

(i) 3 (ii) 8

(iii) 6 (iv) 2.

(d) Let G be a connected graph with 10 vertices. If G is a tree, then the sum of the degrees of the vertices is

(i) 10 (ii) 18

(iii) 20 (iv) 22.

(e) The maximum number of edges in a simple disconnected graph G with 10 vertices and 5 components is

(i) 10 (ii) 15

(iii) 25 (iv) 50.

(f) The maximum and minimum heights of a binary tree with 25 vertices are respectively

(i) 13, 12 (ii) 15, 10

(iii) 17, 4 (iv) 12, 4.

(g) The remainder when $2\times 98!$ is divided by 101 is

(i) 0 (ii) 1

(iii) 97 (iv) 100.

(h) A graph has 15 vertices and 20 edges. The least number of edges to be removed from the graph to make it a tree is

(i) 13 (ii) 5

(iii) 19 (iv) 6.

(i) What is $\sigma(180)$?

(i) 542 (ii) 544

(iii) 546 (iv) 548.

(j) Given any five points inside of a square of side 2, then there exists two points within a distance of atmost

(i) 1 (ii) 2

(iii) $\sqrt{2}$ (iv) $\frac{1}{\sqrt{2}}$.

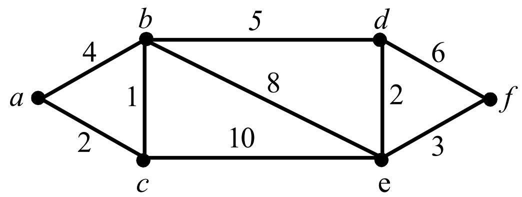
Unit-1

Answer ***any five*** questions.

2. A connected graph contains an open Eulerian trail if and only if it has exactly two vertices of odd degree. — Prove it. 5

3. If G is a connected planar graph with $n(\geq 3)$ vertices, e edges and no cycle of length 3, then prove that $e \leq 2n-4$. Hence show that $K_{3,3}$ is non-planar. 3+2

4. Using Dijkstra's algorithm, find the shortest path from the vertex a to f in the following graph. 5



5. Prove that the number of vertices in a binary tree is always odd. Find also the number of pendant vertices of a binary tree with n vertices. 2+3

6. Prove that for any graph G with $n > 2$ vertices must have two vertices of same degree. 5

7. (a) Give an example of a partially ordered set $(L, \leq)$ which is a lattice.

(b) Prove that in a bounded distributive lattice $(L, \vee, \wedge)$ if an element $a \in L$ has a complement, then it is unique. 2+3

8. Let $n \in \mathbb{N}$. Then prove that there exist two positive integers a and b such that $n^a - n^b$ is divisible by 10. 5

9. Find the number of edge disjoint Hamiltonian cycle in the complete graph K_{11}. 5

Unit-2

Answer ***any four*** questions.

10. Prove that both the functions Tau (τ) and Sigma (σ) are multiplicative functions. 5

11. If p be prime, then $(p-1)! \equiv -1 \pmod{p}$. 5

12. (a) For any two positive integers a and n with $a > n$, show that $n \mid \varphi(a^n - 1)$.

(b) Find the sum of all positive integers which are less than 2022 and prime to 2022. 2+3

13. Solve the quadratic congruence $x^2 + 7x + 10 \equiv 0 \pmod{11}$. 5

14. A certain integer between 1 and 1000 leaves the remainder 1, 2, 6 when divided by 9, 11, 13 respectively. Find the integer with the help of chinese remainder theorem. 5

15. (a) Prove that for all integer $n > 1$, $n^4 + 4$ is composite.

(b) If a is prime to b, prove that $a + b$ is prime to ab. 2+3

16. (a) Define Fermat's number. Give example.

(b) Prove that product of the first n Fermat's number is $2^{2^n} - 1$, by the method of induction. 2+3