2019

PHYSICS

Paper : PHY-411

(Mathematical Methods)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any five*** questions.

1. (a) Show that the set of 2×2 matrices with zeros on the main diagonal, i.e. $\begin{pmatrix} 0 & a_{12} \\ a_{21} & 0 \end{pmatrix}$, form a subspace of the vector space $M_{2\times 2}$ of all 2×2 matrices.

(b) For the cyclic group G of order 4, identify the invariant subgroup H. Show that a homomorphic mapping exists between the group of cosets of H and G.

(c) Find the inverse Laplace transform of the function $\mathcal{L}(s) = (s^2 + 5)/(s^3 - 9s)$. 2+(2+3)+3

2. (a) Given the basis vectors $\mathbf{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$, $\mathbf{u}_3 = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ for $\mathbb{R}^3$, find an orthonormal set of basis vectors using Gram-Schmidt process.

(b) Find a matrix S that diagonalizes $A = \begin{pmatrix} 3 & -2 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$. How does A transform the vector $\mathbf{v}$ with components (1, 1, 1)? 4+(4+2)

3. (a) G is a set of 2×2 matrices, $\begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$, where a, b, c are real numbers such that $ac \neq 0$.

Prove that G forms a group under matrix multiplication. Prove that the subgroup H of G, with $a = c = 1$ is isomorphic to the group of real numbers $\mathbb{R}$ under addition.

(b) Consider a two parameter group of linear transformations, $T(\alpha_1, \alpha_2)$, such that,

$$x' = T(\alpha_1, \alpha_2)\ x = \alpha_1 x + \alpha_2.$$

Identify the indentity element, inverse element and the product element which is the result of two such successive transformations, i.e. $T(\gamma_1, \gamma_2) = T(\alpha_1, \alpha_2)\ T(\beta_1, \beta_2)$.

(c) Show that a rotational transformation of 3-component vectors about the z-axis through an arbitrary angle θ, may be expressed in terms of a 3×3 traceless Hermitian matrix L_3 as $\exp(i\theta L_3)$. (2+2)+3+3

Please Turn Over

4. (a) Consider the one dimensional wave equation on an infinite interval,

$$\frac{\partial^2}{\partial x^2}u(x,t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}u(x,t), -\infty < x < +\infty \cdot$$

(i) Using Fourier transform, reduce the above equation to an ordinary differential equation.

(ii) Find the solution $u(x, t)$, given the initial conditions, $u = f(x)$ and $\frac{\partial u}{\partial t} = 0$ at $t = 0$ $\cdot$

(b) Find and classify the singular points in $-\infty < x < \infty$ for the Chebyshev equation

$$(1 - x^2)y'' - xy' + n^2y = 0.$$

(c) Show that the Frobenius method fails to give a series solution for the equation $y'' - (1/x^3)y = 0.$

(2+4)+2+2

5. (a) Find the Green function for the equation $d^2y(x)/dx^2 + y(x) + f(x) = 0$, subject to the boundary conditions $y(0) = 0$, $y(\pi/2) = 0$. Hence solve the equation $d^2y(x)/dx^2 + y(x) + 1 = 0$ with these boundary conditions.

(b) If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of the self-adjoint equation

$$\frac{d}{dx}\left(p(x)\frac{dy}{dx}\right) + q(x)y(x) = 0,$$

show that the Wronskian $W[y_1, y_2] = C/p(x)$ where C is a constant. Hence, given the solutior $y_1(x) = 1$ for the Legendre equation $\frac{d}{dx}\left(\left(1-x^2\right)y'\right) + n(n+1)y = 0$ with $n = 0$, obtain the second solution.

(3+2)+(3+2

6. (a) Evaluate by the method of contour integration : $\int\limits_{-\infty}^{\infty} dx\frac{x\sin x}{x^2+4}$.

(b) Expand $f(z) = \frac{1}{z^2(1-z)}$ in Laurent series about $z = 1$ for $0 < |z - 1| < 1$. Identify the residue $f(z)$ for the singular point $z = 1$.

(c) Identify the branch points of the complex function $f(z) = \sqrt{z(z-2i)}$. Show the branch cut of th function.

5+(2+1)+(1+

7. (a) Evaluate by the method of contour integration : $\int\limits_{0}^{\infty} dx\frac{x^{-1/2}}{x+1}$.

(b) The real part of an analytic function $f(z)$ is $\sin x \sin hy - 2xy$. Find the imaginary part. Expr $f(z)$ as function of z.

(c) Locate the singularities of $1/\cosh z$.

5+(3+1)