

2021

## STATISTICS — HONOURS — PRACTICAL

Paper : CC-11P

Full Marks : 30

*The figures in the margin indicate full marks.*

1. A random sample of size 9 is drawn from the distribution with pdf

$$f_{\theta}(x) \propto \frac{x^2}{\theta^3}, -3\theta < x < \theta, \theta > 0 \text{ and}$$

$$f_{\theta}(x) = 0, \text{ otherwise,}$$

and the observations are found to be 10, -30, 14, -45, -34, 7, 12, 11, -13.

Find the maximum likelihood estimate of  $\theta$ . Also find (with justification) the maximum likelihood estimate of the variance for the above distribution. 10

2. The life (in hours) of an electrical component is exponentially distributed with mean  $\theta$ , where  $\theta (> 0)$  is an unknown parameter. For testing the null hypothesis  $H_0 : \theta = 6$  against the alternative  $H_0 : \theta < 6$ , four such components are drawn independently. Under a test rule which rejects the null hypothesis when three or more of these four survive for less than six hours, what is the probability of type I error? Also, find the probability of type II error when  $\theta = 4.2$ .

Draw (on your answer-script) a sketch of the power curve of the above test and comment. 10

3. The distribution of a certain random variable is specified by the probability density function  $f$ , where

$$f(x) = \frac{1}{8}(x - \theta) \text{ if } \theta \leq x \leq \theta + 4, \text{ and } f(x) = 0 \text{ otherwise.}$$

A single observation, say  $X$ , is drawn at random from this distribution.

Find a constant  $t$  such that  $[X - t, X + 2]$  is a confidence interval for  $\theta$  with confidence coefficient 0.995. Suggest (with justification) two more confidence intervals for  $\theta$  with confidence coefficient 0.995. Find the expected lengths of the three confidence intervals and comment. 10