M. Sc. (Physics) 4th Semester Examination 2019
PHY 521 (Advanced Condensed Matter I)

Answer in your own words as far as practicable. The marks on the right-hand margin indicate the full marks for the question.

Full Marks: 50 Time: 2hrs

Answer Q. 1 and any three from the rest

1. Answer any five: $5 \times 4 = 20$

(a) Write down the Landau-Ginzburg free energy functional for superconductor metal phase transition in presence of magnetic field. Hence, obtain London's equation. 1+3

(b) How one can distinguish localized state from delocalized state acording to Thouless? Using this concept obtain an expression for the dimensionless conductance for extended states in an arbitrary d dimension of system as a function of its size L. 1+3

(c) Establish that in a one dimensional Ising Model long range order is not possible at a non-zero temperature. Is the result valid in two dimensions? Explain. 2+2

(d) Suppose that the Hamiltonian for N non-interacting fermions is given by

$$H = \sum_i h(r_i)$$

where $h(r_i)$ is the one-particle operator acting on the i-th single particle state. Using a determinental wave function ψ composed of the single particle wave functions ψ_i , evaluate the expectation value $\langle\psi|H|\psi\rangle$. 4

(e) The Hartree-Fock ground state energy for a system of N electrons is given by

$$E_0 = N\Big(\frac{3}{5}\frac{\hbar^2 k_F^2}{2m} - \frac{3}{4}\frac{e^2 k_F}{\pi}\Big)$$

Show that at a sufficiently low density, a spin-polarized ferromagnetic state of electrons becomes more stable than its usual unpolarized counterpart. 4

(f) There is a neutral plasma in equilibrium, consisting of a gas of positively charged ions and electrons. An additional positive charge, oscillating with a frequency ω, is introduced in it. Show that the induced negative charge density $\rho(r)$ performs an oscillation with the same frequency ω, and there is a resonance if

$$\omega = \sqrt{\frac{4\pi n_0 e^2}{m}}$$

Here n_0 is the equilibrium density of electrons. 3+1

2. (a) Show that under the mean-field approximation, the on-site iteraction term in the Hubbard Hamiltonian can be written in the form :

$$U\Big(n_{i\uparrow}\langle n_{i\downarrow}\rangle + n_{i\downarrow}\langle n_{i\uparrow}\rangle - \langle n_{i\uparrow}\rangle\langle n_{i\downarrow}\rangle\Big)$$

How can you incorporate a ferromagnetic order in this expression?

(b) Helical magnetic order occurs in rare earth metals of hcp layered crystalline structure with stacking sequences ABAB.... What kind of interaction and conditions between layers are responsible for such order? Let us assume a negative coupling between next nearest layers exhibing a magnitude of $J_1\sqrt{3}/6$ with J_1 being the coupling strength between adjacent layers. Is this material helimagnetic? Explain.

(4+1)+(2+2+1)

3. (a) Let the wave-function for a many-body fermionic state be given by $|1101100100\cdots\rangle$. Express this state in terms of excitations about the filled Fermi sea $|111110000\cdots\rangle$. Interpret your answer in terms of electron and hole excitations.

(b) Consider a system of N electrons. Instead of the Coulomb interaction, the electrons interact with each other via a repulsive interaction of the form $g\delta(r_i - r_j)\delta_{\sigma_i\sigma_j}$. Here r_i and σ_i are the space coordinate and spin variable of the i-th electron . Express the kinetic energy and potential energy parts of the Hamiltonian in momentum-space representation, in terms of the creation and annihilation operators $c^\dagger_{k\sigma}$ and $c_{k\sigma}$.

(c) What is Kondo effect in a metal? What is the basic scattering processes responsible for such a behaviour? 2+(2+3)+(1+2)

4. (a) Assuming that the superfluid wave function has the form $\psi(r) = \sqrt{n(r)}e^{i\theta(r)}$, prove that $\vec{\nabla} \times \vec{v_s} = 0$, v_s being the superfluid velocity. Considering a flow across a closed tube, show that the circulation is quantized.

(b) A non-interacting Bose gas follows the dispersion relation $\epsilon(p) = p^2/2m$, while, for a weakly interacting Bose gas it is $\epsilon(p) \approx p^2/2m + gn$. Here n is the particle density, and g is the interaction strength. Using Landau's criterion for superfluidity, find out which one, among these two systems, can support a superflow. Find the corresponding critical velocity. (3+3)+4

5. (a) Explain clearly why variational scheme is adopted for calculating the binding energy of the Cooper pairs. The energy difference (W) between the superconducting and normal state is given by

$$W = \sum_k |\xi_k| - \sum_k \frac{\xi_k^2}{E_k} - \frac{\Delta^2}{2} \sum_k \frac{1}{E_k}, \quad \xi_k = \epsilon_k - \mu, \quad E_k = \sqrt{\xi_k^2 + \Delta^2}$$

Show that in the continuum limit with $|\xi_k| \leq \hbar\omega_D$ under weak coupling approximation, W reduces to $-\frac{1}{2}N(E_F)\Delta^2$, where $N(E_F)$ is the DOS at the Fermi energy.

(b) Calculate the magnetization $M(x)$ in an external magnetic field $\vec{B}_0 = B_0\vec{e}_z$ of a superconducting slab of thickness $a \ll \lambda$, where λ is the London penetration depth. Sketch the Magnetization $M(x)$ as a function of x.

(c) Write down the variational wavefunction used in BCS theory. Write down two important characteristic features of high temperature superconductors. (1+3)+(2+1)+(1+2)

6. (a) Explain briefly Mott's minimum metallic conductivity at the mobility edge. What is the numerical value of this conductivty in two dimensions?

(b) What do you mean by variable range hopping (VRH)? Stating clearly the assumptions, derive Mott's VRH conduction formula for an amorphous system in an arbitrary dimension d.

(c) Obtain the density of states of the system described by the Hamiltonian

$$H = -t\sum_i \left(c_{i+2}^\dagger c_i + h.c.\right) + V\sum_i c_i^\dagger c_i$$

defined on a one dimensional lattice (of lattice constant a). Here t and V are constants.

(2+1)+(1+3)+3)