# M. Sc. (Physics) 4th Semester Examination 2019
## PHY 521 (Nuclear Structure)

*Answer in your own words as far as practicable. The marks on the right-hand margin indicate the full marks for the question.*

ull Marks: 50

Time: 2 Hours

**Answer Question No. 1 and any *three* from the rest**

1. Answer **any five** questions:

$4 \times 5 = 20$

(a) Let $\Phi_{\{A\}}(\{\mathbf{x_i}\})$ and $\Phi_{\{B\}}(\{\mathbf{x_i}\})$ be two normalized antisymmetric total wavefunctions of a free $N$-fermion system. $\{A\}$ and $\{B\}$ denote the set of single particle states and $\{\mathbf{x_i}\}$ the set of space and spin coordinates. Evaluate $\int d\tau\, \Phi^*_{\{A\}}\, \hat{F}\, \Phi_{\{B\}}$ where $\hat{F}$ is a symmetric operator and $d\tau$ the volume element corresponding to the coordinates $\{\mathbf{x_i}\}$. 4

(b) Evaluate $\langle \Psi | \hat{N} | \Psi \rangle$, where $\Psi$ is the BCS ground state defined as,

$$|\Psi\rangle = \prod_{k>0}^{\infty} (u_k + v_k c_k^\dagger c_{-k}^\dagger)|0\rangle \text{ and } \hat{N} = (c_k^\dagger c_k + c_{-k}^\dagger c_{-k}).$$

4

(c) The energies of the lowest $1/2^+$ and $3/2^+$ excited states in $^{17}$O with respect to its ground state are 0.87 MeV and 5.08 MeV, respectively. Using this data, explain the importance of inclusion of the $l^2$ term in the Hamiltonian. 4

(d) For a pure harmonic vibrator, what are the states that will be obtained for number of phonons less than or equal to 3? Why are the observed states with the same phonon numbers nondegenerate? 3+1

(e) Assuming a deformed harmonic oscillator potential where $\omega_x = \omega_y \neq \omega_z$, obtain the asymptotic quantum numbers for nucleon numbers between 0 and 8. 4

(f) Determine the most probable types and multipolarities for the following gamma transitions:
i) $1^+ \longrightarrow 0^+$
ii) $3/2^- \longrightarrow 3/2^+$

2+2

2. (a) Consider a two neutron configuration in the shell model. Construct antisymmetrized states with good angular momenta when the two particles are in (i) different single particle states; (ii) the same single particle state.

(b) In the low energy spectrum of the nucleus $^{18}_{8}$O, what are the different spin parity states that are likely to be found? Assume that the neutrons can be placed in the $1d_{5/2}$ and $2s_{1/2}$ orbits.

(c) In the shell model, a particular $J^\pi$ state can be constructed from two configurations. Show that in presence of configuration mixing, the two mixed states are always non-degenerate.

(d) Explain what is meant by the coefficient of fractional parentage considering the case of $^{19}$O, with three neutrons in the state $1d_{5/2}$.

3+2+2+3

3. (a) Find out the magic numbers in a three dimensional harmonic oscillator well potential. Explain the importance of the spin parity term in nuclear shell model.

(b) A fermion system is described by

$$H = H_0 + V = \sum_{k=1}^{N} -\frac{\hbar^2}{2m}\nabla_k^2 + \sum_{k,l=1}^{N} \frac{1}{2}v(|\mathbf{x_k} - \mathbf{x_l}|)$$

Consider two total wave functions $\Psi_{\{A\}}(\mathbf{x_k})$ and $\Psi_{\{B\}}(\mathbf{x_k})$ of $H_0$, where $\{A\}$ and $\{B\}$ denote the set of $N$-single particle states.
Evaluate the matrix elements $\langle\Psi_{\{A\}}|H|\Psi_{\{B\}}\rangle$, when (i) every state in $\{A\}$ is the same as every state in $\{B\}$, (ii) the set of states in $\{A\}$ and $\{B\}$ differ in one state.

(3+3)+(2+2)

4. (a) Write down an expression describing the surface of a deformed nucleus for pure quadrupole deformation. Show that only two parameters are sufficient to describe quadrupole deformation.

(b) Show that the dipole mode cannot be considered as deformation.

(c) Show that the energy values in an axially symmetric rotor are given by the relation

$$E_{I,K} = \frac{I(I+1) - K^2}{2\Theta}$$

, where $I$ and $K$ are the usual quantum numbers for rotational states. Find out the allowed I values for $K = 0$ and $K = 1$.

(1+2)+3+(2+2)

5. (a) Obtain the Hamiltonian for the electromagnetic field in terms of the field amplitude operators $q_\lambda$ and $q_\lambda^\dagger$. Write down the terms describing the interaction between the nucleons and the electromagnetic field.

(b) Estimate the single particle value for B(E$\lambda$) for transition from $J = \lambda + 1/2$ to $J = 1/2$. Assume that the nuclear density is constant inside the nucleus.

(5+2)+3

6. Consider a system of seven fermions, weakly interacting through a two-body potential. The energy levels available to the system, in order of increasing energy, are labeled as $1a$, $1b$, $1c$, $1d$, $1e$, $1f$, $2a$, $2b$, $2c$ . . . etc. Each level can accommodate two fermions of opposing spins.

(a) Construct the lowest state of the system from the vaccuum state $|0\rangle$.

(b) Derive the Hartree-Fock equations for the system in the occupation number formalism.

(c) Obtain explicit expressions for the Hartree and exchange terms and comment on their physical significance.

2+5+3