2021

MATHEMATICS — HONOURS

Paper : DSE-B-1

(Linear Programming and Game Thoery)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***all*** questions with proper explanation / justification (**one** mark for correct answer and **one** mark for justification) : 2×10

(a) Let $x = \{(x, y)|x^2 + y^2 = 1\}$ and y is the set of all convex combinations of the vertices of a cube. Then

(i) x is a convex polyhedron, but y is not.

(ii) x is not a convex polyhedron, but y is a convex polyhedron.

(iii) both x and y are convex polyhedrons.

(iv) neither x nor y is a convex polyhedron.

(b) The number of extreme points of the convex set

$S = \{(x, y) : |x| \leq 1, |y| \leq 1\}$ is

(i) 0 (ii) 2

(iii) 4 (iv) infinitely many.

(c) For the system of equations

$$2x_1 - x_2 + 3x_3 = 3$$

$$-6x_1 + 3x_2 + 7x_3 = -9$$

$x_1 = \frac{3}{2}, x_2 = 0, x_3 = 0$ is

(i) basic feasible but non-degenerate solution

(ii) basic feasible but degenerate solution

(iii) non-basic feasible solution but degenerate

(iv) non-basic, non-degenerate solution.

(d) Consider an L.P.P

Maximize $z = cx$,

subject to the constraints

$$Ax = b, \ x \geq 0$$

(The symbols have their usual meaning).

Then the problem admits of an unbounded solution, if at any iteration of the simplex algorithm,

(i) at least one index number is found to be negative and all elements in the column corresponding to that negative index are non-positive.

(ii) at least one index number is found to be negative and all elements in the column corresponding to that negative index are all positive.

(iii) at least one index number is found to be positive and all elements in the column corresponding to that positive index are non-positive.

(iv) at least one index number is found to be positive and all elements in the column corresponding to that positive index are positive.

(e) $z = 20x_1 + 9x_2$

Subject to $2x_1 + 2x_2 \geq 36$

$6x_1 + x_2 \geq 60$

$x_1 \geq 0, \ x_2 \geq 0$

The minimum value of z is

(i) 360 at (18, 0) (ii) 336 at (6, 4)

(iii) 540 at (0, 60) (iv) 0 at (0, 0).

(f) In solving the L.P.P.

Min $z = 6x + 10y$

Subject to : $2x + y \geq 10$

$x \geq 6$

$y \geq 2$,

$x \geq 0, \ y \geq 0$.

redundant constraints are

(i) $x \geq 6, \ y \geq 2$ (ii) $2x + y \geq 10, \ x \geq 0, \ y \geq 0$

(iii) $x \geq 6$ (iv) none of these.

(g) A degenerate BFS in a balanced TP with m origins and n destinations will consist of

(i) at least $(m + n - 1)$ positive-variables

(ii) at most $mn - (m + n - 1)$ positive variables

(iii) at most $m + n - 1$ positive variables

(iv) at most $m + n - 2$ positive variables.

(h) The assignment problem will have alternate solutions when

(i) total opportunity cost matrix has at least one zero in each row and column.

(ii) the total opportunity cost matrix has at least two zeros in each row and column.

(iii) there is a tie between zero opportunity cost cells.

(iv) two diagonal elements are zeros.

(i) Consider the game with the pay off matrix :

$$\text{Player A} \quad \overset{\text{Player B}}{\begin{bmatrix} p & 7 & 3 \\ -2 & p & -8 \\ -3 & 4 & p \end{bmatrix}}$$

The value of p for which the game is strictly determinable satisfies

(i) $-8 \leq p \leq -3$

(ii) $-3 \leq p \leq -2$

(iii) $-2 \leq p \leq 3$

(iv) $-8 \leq p \leq 7$.

(j) Consider the following pay-off matrix of a game. Identify the dominance in it.

$$\begin{array}{ccc} & & \text{B} \\ & & \begin{array}{ccc} X & Y & Z \end{array} \\ \text{A} & \begin{array}{c} P \\ Q \\ R \end{array} & \begin{bmatrix} 1 & 7 & 3 \\ 5 & 6 & 4 \\ 7 & 2 & 0 \end{bmatrix} \end{array}$$

(i) P dominates Q

(ii) Y dominates Z

(iii) Q dominates R

(iv) Z dominates Y.

Unit - I

2. Answer ***any two*** questions :

(a) A pharmaceutical firm produces two products A and B. Each unit of product A requires 3 hrs. of operation–I and 4 hrs. of operation-II, while each unit of product B requires 4 hrs. of operation-I and 5 hrs. of operation-II. Total time available for operation I and II are 20 hrs. and 26 hrs. respectively. Product A sells at a profit of ₹ 10 per unit, while product B sells at a profit of ₹ 20 per unit. Formulate the problem as an LPP to maximize the profit. 5

(b) Find the basic solutions of the system

$$2x_1 + 4x_2 - 2x_3 = 10$$
$$10x_1 + 3x_2 + 7x_3 = 18$$
$$x_1,\ x_2,\ x_3 \geq 0.$$

Which of them are feasible? Mention the degenerate b.f.s., if there be any. 3+1+1

(c) Define a convex set. Also define a convex polyhedron.

Test if the set $S = \{(x_1, x_2, x_3) : 2x_1 - x_2 + x_3 \leq 4\} \subset R^3$ is a convex set or not. 1+1+3

(d) Prove that the objective function of an L.P.P. assumes its optimal value at an extreme point of the convex set of feasible solutions. 5

Unit - II

3. Answer ***any one*** question :

(a) (i) Apply simplex method to show that the LPP.

Maximize $z = 4x_1 + 14x_2$

Subject to $2x_1 + 7x_2 \leq 21$

$7x_1 + 2x_2 \leq 21$

$x_1,\ x_2 \geq 0$

admits of an alternative optimal solution. Indentify the type of it.

(ii) Use the method of penalty to

Maximize $z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4,\quad x_1, x_2 \geq 0.$ (5+1)+4

(b) (i) Show that the L.P.P.

Maximize $z = 3x_1 + 2x_2$

Subject to $2x_1 + x_2 \leq 2$

$3x_1 + 4x_2 \geq 12$

$x_1,\ x_2 \geq 0$

has no feasible solution.

(ii) Is there any degeneracy in the following LPP?

Minimize $z = 4x_1 + x_2$

Subject to $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 3,\ x_1,\ x_2 \geq 0$

If it is so, resolve that degeneracy and solve the problem. 4+6

Unit - III

4. Answer ***any one*** question :

(a) (i) State and prove the fundamental theorem of Duality.

(ii) Find the dual of the LPP :

Maximize $z = x_1 - x_2 + 3x_3 + 2x_4$

Subject to $x_1 + x_2 \geq -1,\ x_1 - 3x_2 - x_3 \leq 7,$

$x_1 + x_3 - 3x_4 = -2,\ x_1,\ x_4 \geq 0,$

$x_2,\ x_3$ unrestricted in sign. 2+4+4

(b) Use duality to find the optimal solution, if any, of the following LPP :

Maximize $z = 3x_1 + 2x_2$

Subject to $x_1 + x_2 \geq 1$

$x_1 + x_2 \leq 7$

$x_1 + 2x_2 \leq 10$

$x_{2,} \leq 3$

$x_1,\ x_2 \geq 0.$ 10

Unit - IV

5. Answer ***any three*** questions.

(a) Solve the following transportation problem : 5

	A	B	C	D	E	
I	5	8	6	6	3	8
II	4	7	7	6	6	5
III	8	4	6	6	3	9
	4	4	5	4	8	

(b) State the mathematical formulation of a general transportation problem. Also show that an assignment problem is a special case of transportation problem. 3+2

(c) A salesman has to visit five cities A, B, C, D, E. The distances (in hundred kilometers) between the cities are as follows :

	A	B	C	D	E
A	∞	14	12	16	8
B	14	∞	16	10	12
C	12	16	∞	18	14
D	16	10	18	∞	16
E	8	12	14	16	∞

If the salesman starts from city A and has to come back at city A, which route should he select so that the total distance travelled is minimum. 5

(d) Solve the following 2×5 game graphically.

Player B

$$\begin{array}{cc} & \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 2 & -1 & 5 & -2 & 6 \\ -2 & 4 & -3 & 1 & 0 \end{bmatrix} \end{array}$$

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(e) Using dominance, solve the following game :

Player B

$$\begin{array}{cc} & \begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} 2 & -2 & 4 & 1 \\ 6 & 1 & 12 & 3 \\ -3 & 2 & 0 & 6 \\ 2 & -3 & 7 & 7 \end{bmatrix} \end{array}$$

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(f) (i) Write the standard form of LPP corresponding to the following game problem from the point of view of the player B :

Player B

$$\begin{array}{cc} & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} -1 & 1 & 1 \\ 2 & -2 & 2 \\ 3 & 3 & -3 \end{bmatrix} \end{array}$$

(ii) Solve the above problem by simplex method. 1+4