M.Sc. (Physics) 4th Semester 2020
PHY 522 (Particle Physics)

Full marks: 50 Time: 2.5 hrs

(*2 hours for answering and 30 minutes for downloading, scanning, and mailing back*)

Answer any *five* questions.

Instructions:

(a) Write your *Examination Roll Number* and *Registration Number* (from an earlier admit card) at the top of your answer script.

(b) Do not write your name or class roll number anywhere.

(c) Write page number on top of each page.

(d) Scan the complete answer script into *a single pdf file* and mail it to the e-mail from where you got this question paper.

(e) The answer script file must be named as XXXXXXPHYAAA.pdf, where XXXXXX is the university roll number and AAA is the paper identifier. For example, for this paper, the answer script coming from C91/PHS/181099 must be named 181099PHY522.pdf

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1. In a scattering process, there are two particles in the initial state.

   a) What can be said about the number and nature of the final state particles if the scattering is elastic? **(2 marks)**

   b) The deep inelastic scattering experiment to probe the structure of nucleons was designed in analogy with the Rutherford experiement which established the existence of atomic nucleus. Was Rutherford scattering elastic or inelastic? **(2 marks)**

   c) Rutherford analyzed the alpha-scattering experiment by using classical non-relativistic scattering theory. Explain why he did not face any problem for not using relativistic physics. **(3 marks)**

   d) Now argue that the use of relativistic scattering theory was essential for the analysis of deep inelastic electron-proton scattering. **(3 marks)**

   (You need not use very accurate values of the magnitudes of the quantities involved: just order-of-magnitude values will do.)

2. Consider the process

$$\nu_\mu(k) + p(p) \to \mu^-(k') + X \,,$$

where inside the parentheses we have indicated the symbol for the 4-momentum of the particles. X is completely unknown: it need not be a single particle.

a) Draw the simplest diagram contributing to this process, identifying clearly what is the mediating particle. **(2 marks)**

b) What should be the charge, baryon number and lepton number of the combination X? **(1 mark)**

c) What are the possible values of the total angular momentum of X? **(2 marks)**

d) If the initial proton is at rest, what is the minimum energy required for the neutrinos so that this process can take place, where X is just one particle of mass M? (Take proton mass to be m and neglect the masses of the leptons. Do not plug in numerical value of m or anything else.) **(3 marks)**

e) What would be the reaction at the quark level? **(2 marks)**

3. a) Write down the Lagrangian for the interaction between a fermion doublet Ψ and gauge fields W^a_μ in an SU(2) gauge theory with coupling constant g. Consider a theory in which $\Psi = (\nu_{eL} \quad e_L)^T$, W^1_μ and W^2_μ are the usual linear combinations of $W^\pm_\mu$, and W^3_μ is the photon. Obtain all the interaction terms involving the fermions and the gauge bosons. Out of these terms, which one rules out this theory? **(1+3+1 marks)**

b) Consider the decay of a real scalar of mass M into a fermion and an antifermion of mass m through the interaction $h\overline{\psi}\gamma_5\psi\phi$ where h is a coupling constant. Find the constraint on h imposed by the hermiticity of the Lagrangian. (Hint: h can be complex.) Evaluate the modulus squared and spin summed matrix element in terms of M and m. **(1+4 marks)**

4. a) Consider the term $\overline{q}\,\widetilde{\Phi}u_R$ in the Standard Model, where $q = (u_L \quad d_L)^T$, $\widetilde{\Phi} = i\tau_2\Phi^*$ and Φ is the Higgs doublet. Explain why this term is $SU(2)_L$ invariant and $U(1)_Y$ invariant. **(1+2 marks)**

b) Find out whether a term $\Phi^\dagger\widetilde{\Phi} + \widetilde{\Phi}^\dagger\Phi$ is possible in the Standard Model by looking at its $SU(2)_L$ and $U(1)_Y$ transformations. **(2 marks)**

c) Draw all the tree-level Feynman diagrams for the following processes in the Standard Model (you do not need to write down the expressions): (i) $e^+e^- \to \mu^+\mu^-$, (ii) $\nu_e e^- \to \nu_e e^-$. **(3+2 marks)**

5. a) Prove that a vector current is the sum of a left-handed current and a right-handed current:

$$\overline{\psi}\gamma^\mu\psi = \overline{\psi}_L\gamma^\mu\psi_L + \overline{\psi}_R\gamma^\mu\psi_R.$$

(4 marks)

b) What are the helicities of a particle and an antiparicle corresponding to a massless right-handed spinor field ψ_R? **(1 mark)**

c) Assume that nature contains neutrinos and anti-neutrinos of both helicities, and that the leptonic current in the pion decay Lagrangian is the vector current $\bar{e}\gamma^\mu\nu_e$. So the Lagrangian will now contain not only the familiar left-handed current $\bar{e}_L\gamma^\mu\nu_{eL}$ but also $\bar{e}_R\gamma^\mu\nu_{eR}$ (and their hermitian conjugates). Use helicity argument or otherwise to show that for this right-handed current also, the decay $\pi^+ \to e^+\nu_e$ will be suppressed compared to the decay $\pi^+ \to \mu^+\nu_\mu$. **(3 marks)**

d) Write down, with justification, the lepton number and the baryon number of the Higgs boson in the Standard Model. **(2 marks)**

6. a) Relate the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, K, to the mass matrices of the up- and down-type quarks M_u and M_d in the flavour basis. **(3 marks)**

b) If $[M_u, M_d] = 0$ show that the CKM matrix K is an identity matrix. **(1 mark)**

c) Show that for n-quark flavours of the up- and down-type the CKM matrix has $n(n-1)/2$ mixing angles and $(n-1)(n-2)/2$ physical phases. **(3 marks)**

d) For 3-generations of quarks, define $\lambda_u = K_{u\alpha}K^*_{u\beta}$ with $\alpha \neq \beta$ standing for d-type quarks. Using similar expressions for $\lambda_{c,t}$ show that $S = \lambda_u + \lambda_c + \lambda_t = 0$. If instead, we had taken $\alpha = \beta$ what would have been the value of S? **(2+1 marks)**

7. a) What are the different types of neutrinos present in the Standard Model? **(1 mark)**

b) In a two-flavour picture, it is given that the flavour eigenstates ($\nu_\alpha,\ \alpha = 1, 2$) are expressed in terms of the corresponding mass eigenstates ($\nu_i,\ i = 1, 2$) through

$$(\nu_\alpha)_{L,R} = \sum_{i=1}^{2} O_{\alpha i}\ (\nu_i)_{L,R} \qquad \text{with} \qquad O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

where θ is the mixing angle. The neutrino mass matrix is diagonal in the mass basis. Determine the mass matrix in the flavour basis. **(3 marks)**

c) Considering the mixing between ν_e and ν_μ, for a medium of uniform density write down the additional contributions to the two-neutrino mass matrix in the flavour basis. **(3 marks)**

d) Hence show how the neutrino mixing and mass splitting are altered in a medium of uniform density. **(3 marks)**