2019

PHYSICS

Module : PHY–421

(Classical Electrodynamics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any five*** questions.

1. (a) Define spherical moments of a charge distribution. How does the number of independent components of spherical moments differ from that of the Cartesian moments?

   (b) A charge $+Q$ is distributed uniformly along the z axis from $-a$ to $+a$. Show that the electrostatic potential at a point $\vec{r}$ is given by

   $$\Phi(r,\theta)=\frac{Q}{4\pi\epsilon_0 r}\sum_{n=0,2,4,\ldots\ldots}\frac{(a/r)^n P_n(\cos\theta)}{n+1}$$

   (c) Starting from the electromagnetic force on the charges and current, derive the expression for Maxwell stress tensor in the absence of electrostriction and magnetostriction. 3+3+4

2. (a) Calculate the electric and magnetic fields corresponding to the potential $A^{\mu}=K(xyz,-yzt,-xzt,-xyt)$, K being a constant. Does the potential satisfy Coulomb gauge condition? Interpret the results.

   (b) Show that the covariant equation of motion of a particle in an electromagnetic field is

   $$m\frac{\partial u^{\mu}}{\partial\tau}=\frac{e}{c}\sum_{\nu}F^{\mu\nu}u_{\nu}$$

   (c) A classical hydrogen atom has the electron at a radius equal to first Bohr radius at time $t=0$. What is the required time for the radius to decrease to zero due to radiation? What is the life-span of the atom according to this model? 3+3+4

3. Consider a localized source of charges and currents (confined to a very small region of space) which vary sinusoidally in time :

   $$\rho(\vec{r},t)=\rho(\vec{r})\exp(-i\omega t),\ \vec{J}(\vec{r},t)=\vec{J}(\vec{r})\exp(-i\omega t)$$

   (a) Find the vector potential $\vec{A}(\vec{r},t)$ in the Lorentz gauge provided no boundary surfaces are present. Find its form in the radiation zone ($kr\gg 1$, $k=\omega/c$).

(b) Find the electric dipole fields $\vec{E}$ and $\vec{B}$ arising out of the source. Determine the angular distrib of the power radiated and the total power radiated.

(c) Determine the fields $\vec{E}$ and $\vec{B}$ due to the magnetic dipole moment of the source and state these are related to the electric dipole fields.

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4. (a) The action of an electromagnetic field is

$$A = \int \left[-\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c} A^{\mu} J_{\mu} \right] d^4 x \, .$$

Establish the Maxwell's inhomogeneous equations from this action using Lagrangian formula (Gaussian unit is chosen)

(b) Show that above action is invariant under gauge transformation $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\alpha(t, \vec{x})$ provided current J^{μ} is conserved and is confined in the finite part of space-time, where α is any functi space and time.

(c) Using covariant formulation, show that the relativistic generalization of Larmor's formula for arbi velocity of charge is given by

$$P = \frac{1}{4\pi\epsilon_0} \frac{2q^2\gamma^6}{3c} \left[\dot{\beta}^2 - \left(\vec{\beta} \times \dot{\vec{\beta}} \right)^2 \right].$$

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5. (a) Argue that there are only two Lorentz invariants in electromagnetic field. Which one of t is a pseudo-scalar and why?

(b) Indicate clearly the arguments for constructing complete relativistic Lagrangian of a charged par in an electromagnetic field. Assuming the vector potential $\vec{A} = \frac{\mu \sin(\theta)}{r^2} \hat{\phi}$ in spherical polar coord system, obtain an expression of canonical momentum p_{ϕ} conjugate to ϕ. Show that p_{ϕ} is con of motion.

(c) Write down Faraday's equation in terms of field tensor $F_{\mu\nu}$.

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6. (a) A sudden burst of current $I(t) = k\delta(t)$ flows through an infinite straight electrically neutral Determine the retarded vector potential at a distance s from the wire.

(b) A charged harmonic oscillator with the natural frequency of oscillation ω_0 is excited by an ext impulse. The electric field produced by such an oscillating charge has the form

$$\vec{E}(t) = \vec{E}_0 \, e^{-i\omega_0 t} e^{-\gamma t/2}.$$

Calculate the spectral width of radiation.

(c) A charged particle is subjected to a force $F(t) = k\delta(t)$ (for some constant k). Show that t is a discontinuity in acceleration.

(d) Show that retarded solutions satisfy the Lorentz condition.

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7. (a) Starting from the relevant magneto-hydrodynamic equations, derive the equation :

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left(\vec{v} \times \vec{B}\right) + \eta_m \nabla^2 \vec{B}.$$

Hence, calculate the diffusion time and magnetic Reynold number.

(b) Consider cylindrical plasma of radius R with longitudinal current. Show that mechanical pressure satisfies the relation :

$$p(r) = \frac{1}{2\mu_0} \int_r^R \frac{1}{r^2} \frac{d}{dr}\left(B_\phi^2(r) r^2\right) dr,$$

where symbols have their usual meaning. How does the pressure vary for constant current?

(c) Indicate the mechanism of pinch effect and sources of so called sausage and kink instabilities.

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