# M. Sc. (Physics) 3rd Semester Examination 2018
## PHY 514 (Solid State Physics)

*Answer in your own words as far as practicable. The marks on the right-hand margin indicate the full marks for the question.*

*Answer Q. 1 and any three from the rest*

1. Answer any five

(a) Let the periodic potential for an electron in a crystal be given by

$$V = -V_0 \sum_n \delta(x - na)$$

in a one dimensional lattice with lattice constant $a$. Show that energy bands are obtained when the wave-function of the electron is in the Bloch form. 4

(b) Consider a crystal field potential of the form $D(x^4 + y^4 + z^4 - (3/5)r^4)$ for a single electron with angular momentum $L = 1$ moving about a nucleus. Check that the phenomenon of quenching of angular momentum is taking place here in the first order perturbation. Is the degeneracy with respect to the magnetic quantum number lifted? 2+2

(c) A diatomic chain comprises of two types of atoms with mass $M$ and $m$ respectively. Considering small oscillations about some equilibrium positions, show that in the long wavelength limit, the system can oscillate with a nearly constant frequency. 4

(d) Show that for a completely filled band, the total current density is zero when an electric field is applied. Hence show that for a partially filled band, the total current density can be attributed to holes. 2+2

(e) Starting from Bloch's equation in NMR derive an expression for 90° pulse time. Calculate the number of hyperfine components in ESR line for F centers in an fcc crystal with I = 1/2 for nuclear spin quantum number of nuclei of neighbouring cations. How do the spin configurations determine the ESR line shape? 2+1+1

(f) Show that for dipolar dispersion the expression for the loss tangent can be expressed as $D = \frac{1}{\omega CR}$, where $C$ and $R$ are frequency dependent capacitance and resistance respectively. If the activation energy for a dipolar system is 0.5 eV, estimate the frequency at which the peak of $D$ will appear at room temperature. 2+2

2. (a) Consider a di-atomic molecule due to van der Waals bonding.

(i) Show that the equilibrium separation is given by

$$R_0 = 2^{1/6}\sigma$$

where $\sigma$ is the atomic diameter.

(ii) Find the strength of the van der Waals bond in the molecule.

(b) How does Laue formulation of X-ray diffraction by a crystal differ from Bragg formulation? Show that the condition for constructive interference in Laue formulation is

$$\mathbf{d} \cdot (\mathbf{K} - \mathbf{K}') = 2\pi m$$

where, symbols have their usual meaning.

(c) Consider a monoclinic lattice with non-orthogonal angle $\gamma$. The conventional basis vectors are given by

$$\mathbf{a} = a\mathbf{e}_x, \mathbf{b} = b(\cos\gamma\mathbf{e}_x + \sin\gamma\mathbf{e}_y), \mathbf{c} = c\mathbf{e}_z.$$

Within the conventional cell, the atoms are at $(0,0,0)$ and $(\mathbf{a}+\mathbf{c})/2$. Assume $\gamma$ is an acute angle.

(i) Show that the same lattice can be generated using the primitive vectors

$$\mathbf{a_1} = (\mathbf{a}+\mathbf{c})/2, \mathbf{a_2} = \mathbf{b}, \mathbf{a_3} = (\mathbf{a}-\mathbf{c})/2.$$

(iii) Find the volumes of the conventional and primitive unit cells.

(3+4+3)

3. (a) Explain briefly the physical reason behind the formation of a Cooper Pair?

(b)In the two fluid model for a superconductor, the normal electrons obey the Drude-like equation

$$\frac{d\mathbf{j_n}}{dt} = \frac{n_n e^2}{m}\mathbf{E} - \mathbf{j}_n/\tau$$

where, $n_n$ and $\mathbf{j_n}$ are respectively the number and current densities of the normal electrons. Other symbols have their usual meaning.

The superconducting electrons obey London equation.

(i) Find the frequency-dependent complex conductivity $\sigma(\omega)$ for a superconductor. Consider the time dependent part as $e^{-i\omega t}$ and assume that normal and superconducting fluids respond independently to the electric field.

(ii) Show that, in the low frequency limit, the response of the normal fluid is purely ohmic while the response of the superconducting fluid is purely inductive.

(c) Under the assumption that an electron is strongly bound to the periodic crystalline potential $V_{cr}$, show that its wave function can be expressed as a linear combination of atomic orbitals obeying Bloch's theorem.

Writing down the $V_{cr} = V_{atomic} + \Delta V$, find out an expression for the energy of the electron. Which term is responsible for the formation of the bands?

2+3+(2+3)

4. (a) Write down Lyddane-Sachs-Teller (LST) relation indicating different terms within it. Hence discuss its application in reference to the ferroelectric transition.

(b) Landau free energy for ferroelectric transition is expressed as $F = F_0 + \frac{1}{2}\lambda_2 P^2 + \frac{1}{4}\lambda_4 P^4 + \frac{1}{6}\lambda_6 P^6 \ldots$. Establish a relation among the coefficients $\lambda$ in the expression for $F$ for first order transition at $T = T_c$.

(c) Show that in dipolar dispersion, polarizability satisfies the relation $\alpha_d(\omega) = \frac{\alpha_d(0)}{1-i\omega\tau}$, where symbols have their usual meanings. Plot the imaginary part of the dielectric function with real part of the function for Debye and Cole-Cole relaxation processes on the same graph.

(1+2)+3+(3+1)

5. (a) Assuming that all the phonon modes of a crystal with $N$ atoms have the unique frequency $\omega$, find the thermal entropy $S_{th}$. Now consider the presence of $n$ vacancies such that the $x$ neighbours of

a vacancy have a lower frequency of oscillation $\omega'$. Show that the change in the thermal entropy is given by

$$\Delta S_{th} = 3nxk \ln \frac{\omega}{\omega'}.$$

(b) Calculate the specific heat of the pure crystal in three dimensions using the assumption of a unique frequency of oscillation for the phonons and show that in the high temperature limit $C_v = 3Nk$.

(4+2)+(2+2)

6. Consider electrons with mass $m$ in a magnetic field $\mathbf{B}$ which is in the $\hat{z}$ direction. Assume that the electrons are confined in the $x$ and $y$ directions.

(a) Show that the Hamiltonian reduces to that of a one dimensional harmonic oscillator with a shifted origin.

(b) Show that the degeneracy of the Landau energy levels is equal to

$$D = \frac{m\omega_c A}{2\pi\hbar},$$

where $\omega_c$ is the cyclotron frequency and $A$ the area of the two dimensional confining box.

What will happen to the Landau levels at a finite temperature?

(c) Taking the degeneracy $D = \rho B$, find the total energy when the number of electrons is 60, $\rho = 0.4$ and $B = 40$ in appropriate units.

3+(2+2)+3