M. Sc. (Physics) 4th Semester Examination 2019
PHY 523 (Nonlinear Dynamics)

nswer in your own words as far as practicable. The marks on the right-hand margin indicate the full marks for the question.

l Marks : 50

Time: 2 Hours

Answer Q. 1 and any three from the rest

1. Answer any *five*

(a) Find the fixed points and their nature for the system $\dot{x} = 4x^2 - 16$. Plot schematically $x(t)$ as a function of t for $x(0) = -4.0$ and $x(0) = 1.0$. 2+2

(b) Consider the initial value problem $\dot{x} = x^{1/3}$ with $x(0) = 0$. Comment on the existence and uniqueness of the solution(s). What happens for the initial condition $x(0) = 1$? 3+1

(c) Identify graphically, the nature of bifurcation in the system $\dot{x} = \mu - 3x^2$, with parameter μ. Find the associated potential function $V(x)$ and plot it for $\mu < 0$, $\mu > 0$ and $\mu = 0$. 2+2

(d) For the map $f(x, a) = -(1 + a)x + x^2$, define the universal constants δ. Given that the renormalisation equation for the bifurcation point a_k is $a_k = a_{k+1}^2 + 4a_{k+1} - 2$, find the value of δ. 4

(e) Define similarity dimension of a self-similar fractal set and find the dimension of Cantor set. 4

(f) Consider Lorenz equations

$$\begin{aligned} \dot{X} &= -\sigma X + \sigma Y \\ \dot{Y} &= -XZ + rX - Y \\ \dot{Z} &= XY - bZ. \end{aligned}$$

Locate the fixed points and show that one of them is stable for $r < 1$ and unstable for $r > 1$. 4

2. (a) For the linear system $\ddot{x} - x = 0$, find an equation for the trajectories and plot the trajectories in the phase plane. Check if the system is a gradient system and find the associated Lyapunov function if any.

(b) For the nonlinear system

$$\dot{x} = x - y \qquad \dot{y} = 1 - x\,y$$

classify the fixed point(s) using the Jacobian matrix (i.e. linearization). Draw the phase portrait schematically. (2+2)+(3+3)

3. (a) What is a limit cycle ? Explain briefly why limit cycles are observed only in nonlinear systems of order (dimension) > 1.

(b) Make a phase plane analysis of the van der Pol oscillator,

$$\ddot{x} + \mu\,(x^2 - 1)\dot{x} + x = 0\,,$$

in the strong nonlinear limit, $\mu >> 1$, using the variables,

$$w = \dot{x} + \mu\,F(x) \quad \text{and} \quad y = w/\mu, \quad \text{where } F(x) = \int_0^x du(u^2 - 1).$$

(c) Plot $x(t)$ as a function of t clearly indicating different time scales.

(d) Estimate the time period of the oscillator.

(1+2)+4+1+2

4. (a) For the following system expressed in polar coordinates,

$$\dot{r} = r(1-r^2) + \epsilon\, r\cos\theta \quad \text{and} \quad \dot{\theta} = 1\,,$$

show that a stable limit cycle exists at $r = 1$ for $\epsilon = 0$.

(b) For positive yet sufficiently small ϵ, find a trapping region in the two dimensional phase plane and hence establish the existence of a closed orbit using Poincare-Bendixson Theorem. *Hint : Identify inner and outer radii, r_1 and r_2, of an annular region, such that $\dot{r} > 0$ for r_1 and $\dot{r} < 0$ for r_2.*

(c) For the logistic map $f(x) = rx(1-x)$, locate the 1-cycle fixed points and the 2-cycle fixed points.

2+3+(2+3)

5. (a) For Rayleigh-Benard convection, derive the equation for continuity and the equation for heat conduction. Also, write down the Navier-Stokes equation. What is Boussinesq approximation? How are the three equations modified under this approximation?

(b) Describe briefly how a bifurcation occurs in the plot of velocity vs. temperature difference for roll convection.

(2+2+1+2)+3

6. (a) Locate and analyse the stability of the fixed points for the map $f(x) = \sin x$.

(b) Consider the Henon map,

$$\begin{aligned} x_{n+1} &= 2x_n^2 + 2Cx_n - y_n \\ y_{n+1} &= x_n \end{aligned}$$

where C is a parameter. (i) Show that this map is area-preserving. (ii) Find the (1-cycle) fixed points. (iii) Find the regions of values of C for which the fixed points are stable.

2+(1+1+6)