2021

STATISTICS — HONOURS

Second Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

(Linear Algebra)

(Marks : 25)

Answer *any two* from *question nos.* **1** to **4** and *any one* from *question nos.* **5** and **6**.

1. Show that $f_1(t) = 1$, $f_2(t) = t - 2$, and $f_3(t) = (t - 2)^2$ form a basis of P_3. Express $3t^2 - 5t + 4$ as a linear combination of f_1, f_2 and f_3.   5

2. If $\underset{\sim}{u}$ and $\underset{\sim}{v}$ are column vectors with n and m components respectively and if $\underset{\sim}{u} \neq \underset{\sim}{0}$, show that there exists an $m \times n$ matrix A such that $A\underset{\sim}{u} = \underset{\sim}{v}$.   5

3. Prove that a quadratic form $\underset{\sim}{x}'A\underset{\sim}{x}$ can be written as the product of two linearly independent linear forms in $\underset{\sim}{x}$ iff A has rank 2 and signature o.   5

4. Prove the following without expanding the determinant :

$$\begin{vmatrix} 1 & 1 & 1 \\ x & x & x \\ yz & xz & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x & x & x \\ x^2 & y^2 & z^2 \end{vmatrix}$$

    5

5. (a) Show that a square matrix A is non-singular iff $|A| \neq 0$.

   (b) Solve the following system using Cramer's rule :

$$2x_1 - x_2 + x_3 = -3$$
$$x_1 + x_2 - 3x_3 = 17$$
$$5x_1 - 2x_2 - 4x_3 = 20$$

   (c) Using Cramer's rule, find the value of x_4 in the solution of $A\underset{\sim}{x} = \underset{\sim}{b}$, where

$$A = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ -2 & 3 & 1 & 3 \end{vmatrix} \text{ and } \underset{\sim}{b} = (2, 4, -1, -4)^1$$

    5+5+5

Please Turn Over

6. (a) If A is a n.n.d. matrix of order n with rank r and if $k \geq r$, prove that there exists an $n \times k$ matrix C such that $A = CC'$.

   (b) Prove that every square matrix is a product of triangular matrices.   7+8

(Population Statistics)

(Marks : 25)

Answer *any two* from *question nos.* **7 to 10** and *any one* from *question nos.* **11** and **12**.

7. Explain the purpose and procedure for standardizing death rates.   5

8. Define force of mortality. If $\mu_x = A + BC^x$, find the expression for l_x.   5

9. Differentiate between population projection and population estimation.   5

10. Define NRR and GRR. Why is GRR considered as the upper limit of NRR?   5

11. Derive the equation to a logistic curve stating clearly all the assumptions. Find the estimates of the concerned parameters. How are they obtained using the decennial population data of a country? Is this curve suitable for representing the growth of Indian population?   5+5+3+2

12. (a) Describe the various components of an abridged life table stating the interrelations between them.

    (b) Prove that :   9+(3+3)

    (i) $\displaystyle {}_nq_x = \frac{d_{n+x-1}}{l_x}$

    (ii) $\displaystyle p_x = \frac{e_x}{1 + e_{x+1}}$.