## 2022

## MATHEMATICS - GENERAL

## Paper : GE/CC-4

## Full Marks : 65

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## Group - A

1. Choose the correct answer:
(a) Which of the following set is a group with respect to addition
(i) $\{-3,-2,-1,0,1,2,3\}$
(ii) $\{-1,1\}$
(iii) $\{-1,0,1\}$
(iv) $\{0\}$.
(b) -2 is an eigenvalue of the matrix $M=\left(\begin{array}{ccc}2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1\end{array}\right)$. Then $M^{-1}$ has an eigenvalue
(i) -2
(ii) 1
(iii) 2
(iv) $-\frac{1}{2}$
(c) Probability that at least one of the events A and B occurs is
(i) $P(A)+P(B)-P(A B)$
(ii) $P(A)+P(B)+2 P(A B)$
(iii) $P(A)+P(B)+P(A B)$
(iv) $P(A)+P(B)-2 P(A B)$
(d) Number of divisor of zero in the $\operatorname{Ring}\left(\mathbb{Z}_{5}, \oplus, \odot\right)$ is
(i) 0
(ii) 1
(iii) 2
(iv) 3
(e) If $(0,1,3)=a(2,1,1)+b(4,2,2)$, then the values of $a$ and $b$ are
(i) $(1,1)$
(ii) $(-1,1)$
(iii) $(0,0)$
(iv) None of these.
(f) For the probability density function given by $f(x)=\left\{\begin{array}{cc}e^{-x}, & x \geq 0 \\ 0, & \text { elsewhere }\end{array}\right.$ the mean is
(i) 1
(ii) $\frac{1}{2}$
(iii) 2
(iv) 4
(g) If $E\left(T_{1}\right)=\theta_{1}+\theta_{2}$ and $E\left(T_{2}\right)=\theta_{1}-\theta_{2}$, then the unbiased estimator of $\theta_{1}$ is
(i) $T_{1}+T_{2}$
(ii) $\frac{1}{2}\left(T_{1}-T_{2}\right)$
(iii) $\frac{1}{2}\left(T_{1}+T_{2}\right)$
(iv) $\frac{1}{2}\left(T_{2}-T_{1}\right)$
(h) Binary number corresponding to the decimal number 27.625 is
(i) 11011.101
(ii) 10111.101
(iii) 11101.011
(iv) 11011.011
(i) Which of the following can be a variable name in C ?
(i) Volatile
(ii) True
(iii) Friend
(iv) Export.
(j) The value of the FORTRAN expression : $\left(A^{*}(B+C)\right) / D+A$, where $A=3, B=5, C=-2$ and $D=4$ is
(i) 3
(ii) 4
(iii) 5
(iv) 6

## Group-B

## Unit-1

(Algebra - II)
2. Answer any three questions:
$5 \times 3$
(a) Prove that the set $Q \backslash\{-1\}$ is a group with respect to the composition ' $o$ ' defined by $a o b=a+b+a b$. Is it abelian?
(b) Show that the ring of matrix $\left[\left(\begin{array}{cc}a & b \\ 2 b & a\end{array}\right): a, b \in \mathbb{R}\right]$ does not form a Field, $\mathbb{R}$ being the set of all real numbers.
(c) Is the set $U=\left\{(x, y, z) \in \mathbb{R}^{3}: x-2 y+3 z=0\right\}$ a subspace of the real vector space $\mathbb{R}^{3}$ ? If so, find the basis and dimension of this subspace.
(d) Find the eigenvalues and eigenvectors of the matrix $\left(\begin{array}{ccc}1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6\end{array}\right)$.
(e) Show that the real quadratic form $5 x^{2}+y^{2}+14 z^{2}-4 y z-10 z x$ is positive definite.

## Unit-2

## (Computer Science and Programming)

3. Answer any four questions :
(a) Find the product of $(11.0011)_{2}$ and $(10.01)_{2}$ and also find the octal and hexadecimal equivalents of the product.
(b) Draw a flowchart for computing the g.c.d. of two positive integers $m$ and $n$.
(c) (i) Let $\mathrm{A}=2.7, \mathrm{~B}=3.5$ and $\mathrm{L}=\mathrm{ABS}\left(\mathrm{A}-3 .{ }^{*} \mathrm{~B}\right) / 5$. Find what will be stored at L .
(ii) Write FORTRAN expression of $\frac{\sqrt{a+\log _{e} b}}{c+d \sin x}$
(d) Write an algorithm to sort $n$ given integers in descending order.
(e) Write a FORTRAN program to find the area of a triangle whose three sides are given.
(f) What is positional number system? Why are binary numbers used in computer design? $2+3$
(g) Write a FORTRAN program to check whether a year is a Leap year or not.

## Unit-3

(Probability and Statistics)
4. Answer any four questions:
(a) Bag $A$ contains 2 white and 3 red balls; and bag $B$ contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from bag $B$.
(b) Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Show that the chances that exactly two of them will be children is $\frac{10}{21}$.
(c) Find the coefficient of correlation from the following data :

| $x$ | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $f$ | 2 | 3 | 5 | 10 | 5 |

(d) Draw a Histogram from the following distribution :

| Age Group | $14-15$ | $16-17$ | $18-20$ | $21-24$ | $25-29$ | $30-34$ | $35-39$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of wage earners | 60 | 140 | 150 | 110 | 110 | 100 | 90 |

Please Turn Over
(e) The population of scores of 10 years children in a test is known to have a standard deviation 5.2. If a random sample of size 20 shows a mean of 16.9 , find $95 \%$ confidence interval for the mean score of the population, assuming that the population is normal.
(Given that $\left.\frac{1}{\sqrt{2 \pi}} \int_{1.96}^{\infty} e^{-\frac{x^{2}}{2}} d x=0.025\right)$.
(f) If the equations of two regression lines obtained in a correlation analysis are $2 y+x=11$ and $2 x+3 y-18=0$, determine which one of them is the regression equation of $x$ on $y$. Find the means and correlation coefficient of $x$ and $y$.
(g) In a random sample of size 400 there are 80 defective items. Test at $5 \%$ level whether the proportion of defective items in the population may be regarded as $\frac{1}{6}$.

$$
\left[\text { Given } \int_{0}^{1.96} \phi(t) d t=0.475, \phi \text { is the pdf of normal variate }\right]
$$

