M. Sc (Physics) 4th Semester Examination 2019
PHY 522 : Particle Physics

Answer in your own words as far as practicable. The marks on the right hand side denote the full marks for the respective questions.

Full Marks : 50

Time: 2 Hours

Answer **Question No. 1**, and **any three** from the rest

1. Answer any *five*: 5 × 4

(a) Consider the $SU(2)$ gauge invariant Lagrangian

$$\mathcal{L} = \frac{1}{2}(D_\mu\Phi)^T(D_\mu\Phi) - V(\Phi) - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a$$

where Φ is a triplet of real scalar fields and other symbols have their usual meanings. The minimum of $V(\Phi)$ is $\langle\Phi\rangle = (0 \ \ 0 \ \ v)^T$, leading to SSB. Given the $SU(2)$ generators $(T^a)_{bc} = -i\epsilon_{abc}$, show that two of the gauge bosons become massive while one stays massless.

(b) For what limiting value of ξ does the R_ξ gauge propagator

$$D_{\mu\nu}(k) = -\frac{1}{k^2 - M^2}\left[g_{\mu\nu} - \frac{(1-\xi)k_\mu k_\nu}{k^2 - \xi M^2}\right]$$

for the massive gauge boson reduce to the unitary gauge propagator? Write down the expression for the latter. Why the R_ξ gauge propagator is suitable for proving renormalizability but the unitary gauge propagator is not?

(c) Draw all the Feynman diagrams contributing to $e^+e^- \to \mu^+\mu^-$. Why can the Higgs-mediated diagram be neglected? Define forward-backward asymmetry in terms of the differential scattering cross-section (no calculation is needed). Which diagram gives rise to the forward-backward asymmetry?

(d) Given that the π-mesons have intrinsic parity -1, calculate the CP quantum numbers for $\pi^+\pi^-$ and $\pi^+\pi^-\pi^0$ systems for zero angular momentum.

(e) Show that quark-parton model predicts the ratio $F_2^{ep}/F_2^{en} \to 1$ when sea quark contribution dominates over valence quark contribution inside a proton. While in the case of valence quark dominance, $F_2^{ep}/F_2^{en} \to 0.25$.

(f) Calculate the center-of-mass energies of (i) an e^+e^- collider in which e^+ and e^- collide head on with energies 1 TeV and 0.5 TeV respectively (ii) an e^-p collider with e^- energy of 1 TeV and a proton target at rest. You may use $m_p \simeq 1$ GeV.

2. (a) Draw the Feynman digram and write down the invariant matrix element for the decay $\pi^-(q) \to \mu^-(p)\bar{\nu}_\mu(k)$. What is the corresponding Lagrangian? Show that $\overline{|\mathcal{M}|^2} = 4G_F^2 f_\pi^2 m_\mu^2 (p\cdot k)$ if the neutrino is massless. Express $p\cdot k$ in terms of the pion and muon masses.

(b) Use helicity argument to conclude that a neutral pion cannot decay into a neutrino and an anti-neutrino in the Standard Model. [(2+1+4+1)+2]

3. (a) In the GWS model, consider the part $\mathcal{L} = \bar{\Psi}_L i\gamma^\mu D_\mu \Psi_L + \bar{e}_R i\gamma^\mu D_\mu e_R$, where $\Psi_L = (\nu_L \;\; e_L)^T$. Give the expressions for $D_\mu \Psi_L$ and $D_\mu e_R$. Extract the interaction of the electron with the photon, given $Z_\mu = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$, $A_\mu = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$ and $\tan\theta_W = g'/g$. Hence relate the electromagnetic coupling e to g and θ_W.

(b) Consider the potential $V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger\Phi)^2$, where $\mu^2 < 0$, $\lambda > 0$ and Φ is the $SU(2)$ complex Higgs doublet. If $\Phi = (0 \;\; v + H(x))^T$, find the VEV v and the mass of the Higgs field $H(x)$.

[(2+3+1)+(2+2)]

4. (a) Why does ν_e interact differently from ν_μ and ν_τ when neutrinos pass through normal matter? (Give Feynman diagrams, but amplitudes are not needed.)

(b) The $\nu_e \to \nu_\mu$ oscillation probability in vacuum is given by $P = \sin^2(\Delta m^2 L/4E)$ in natural units for the mixing angle $\theta = \pi/4$. This changes to $P = \sin^2(C\Delta m^2 L/E)$ when Δm^2 is in eV^2, L is in Km and E is in GeV. Find the numerical constant C, given $\hbar c = 200$ MeV fm.

(c) Show that the mass term in the Dirac Lagrangian for an electron $\mathcal{L}_{mass} = m_e(\bar{e}_L e_R + \bar{e}_R e_L)$ does not respect $SU(2)_L \times U(1)_Y$ gauge symmetry of the SM. Verify that the term $\lambda(\bar{L}\Phi e_R + H.c)$ is gauge invariant. Here, Φ is the SM Higgs (doublet) field. $L \equiv \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$ and e_R are an $SU(2)$ doublet and singlet respectively.

[2+3+ (2+3)]

5. (a) Show that mean square radius $\langle r^2 \rangle$, of a spherically symmetric charge distribution for small values of momentum transfer, is given by $-6\frac{\partial F(q)}{\partial q^2}$. Here $F(q)$ is the form factor of the charge distribution and $F(q) \equiv \int \rho(\bar{r}) e^{i\bar{q}.\bar{r}} d^3\bar{r}$.

(b) Write down the relation, in the parton model, between the parton distribution functions and the structure functions F_1 and F_2 used to describe deep inelastic electron proton scattering. Establish a relation between F_1 and F_2.

(c) Show that the experimental data on F_2 ($\int F_2^{ep}(x)dx = 0.18$ and $\int F_2^{en}(x)dx = 0.12$) indicates that the valence quarks inside a nucleon carry only half of nucleon's momentum.

[3+3+4]

6. (a) How does CKM mixing matrix in the W-boson interactions arise from the up-type and down-type quark mass matrices in the flavour basis?

(b) Explain how does the unitarity of V_{CKM} lead to so called *Unitarity triangles* in the complex plane? How many independent unitarity triangles can one have ?

(c) Show that the Z-boson or photon interaction with quarks cannot change the flavour of a quark.

[3+(2+2) + 3]