

2021

## STATISTICS — GENERAL

Second Paper

Full Marks : 100

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Group - A

Marks : 50

Answer **question no. 1** and **any three** questions from the rest.

1. Answer **any four** from the following questions : 2×4
- (a) Distinguish between parameter and statistic.
- (b) What is ‘power’ in hypothesis testing?
- (c) If  $x_1, x_2, x_3$  and iid  $N(0,1)$  variates, then write down the pdf of  $T = \frac{x_1\sqrt{2}}{\sqrt{x_2^2 + x_3^2}}$ .
- (d) If  $F \sim F_{n_1, n_2}$ , what will be the distribution of  $F' = \frac{1}{F}$ ?
- (e) If  $x \sim \text{Bin}(1, p)$ , find an unbiased estimator for  $p^2$ .
- (f) Write down the  $100(1 - \alpha)\%$  confidence interval for variance under normal set up with known mean.
- (g) Write down the simplified form of the Pearsonian Chi-square statistic for testing independence of two attributes in  $2 \times 2$  contingency table.
- (h) If  $\theta$  is a parameter and  $T$  is an estimator such that  $E(T) = \frac{\theta}{2}$ , suggest an unbiased estimator of  $\theta$  and another biased estimator based on  $T$ .
2. (a) Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from  $N(\mu, \sigma^2)$  distribution. Find the sampling distribution of  $\sum_{i=1}^n (X_i - \bar{X})^2$ , where  $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ .
- (b) If  $S^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$ , show that  $S^2$  is not an unbiased estimator of  $\sigma^2$ . 10+4

Please Turn Over

3. Let  $x_1, x_2, \dots, x_n$  be a random sample of  $n$  observations from  $N(\mu, \sigma^2)$ . Examine the unbiasedness and consistency of the sample mean as an estimator of  $\mu$ . Derive the maximum likelihood estimators for  $\mu$  and  $\sigma^2$ . 4+4+6
4. (a) Derive a suitable test for testing  $H_0 : \mu_1 = \mu_2$  against all possible alternatives for two independent normal populations  $N(\mu_1, 3^2)$  and  $N(\mu_2, 2^2)$ .  
 (b) Under the above set up, obtain a  $100(1 - \alpha)\%$  confidence interval of  $(\mu_1 - \mu_2)$ . 8+6
5. (a) Obtain the mean and variance of Chi-square distribution with  $m$  degrees of freedom.  
 (b) Describe the use of Pearsonian Chi-square statistic in testing for goodness of fit. (4+4)+6
6. (a) Define type-I error and type-II error in context of testing of hypothesis. Let  $p$  be the probability that a coin will fall head in a single toss. In order to test  $H_0 : p = \frac{1}{2}$  against  $H_1 : p = \frac{2}{3}$ , the coin is tossed 5 times and  $H_0$  is rejected if more than 3 heads are obtained. Find the probabilities of type-I and type-II errors.  
 (b) Describe the method of moments in estimation of parameters. (2+2+3+3)+4
7. Write short notes on *any two* of the following : 7×2
- (a) F distribution  
 (b) Properties of ML estimator  
 (c) Large sample test for equality of proportions of successes for two independent binomial populations.

**Group - B****Marks : 50**Answer *question no. 8* and *any three* questions from the rest.

8. Answer *any four* from the following questions : 2×4
- (a) Distinguish between defect and defective.  
 (b) What do you mean by 'rational subgroup'?  
 (c) Define the curtate expectation of life.  
 (d) What is the crude rate of natural increase?  
 (e) Define Chain index number.  
 (f) What do you mean by price relative?  
 (g) What do you mean by cyclical variations?  
 (h) Is Infant Mortality Rate (IMR) a probability rate in the true sense? Give reasons.
9. (a) Define GRR and NRR. Discuss how good they are as indices of population growth.  
 (b) What do you understand by it : 'The NRR for a Country is 1.327'? (6+4)+4

10. (a) Describe how you would fit —  
(i) a linear trend;  
(ii) an exponential trend to time series data using the method of least squares.  
(b) Discuss the ratio-to-trend method for determining seasonal indices for monthly data. (4+4)+6
11. (a) What are Marshall-Edgeworth, Laspeyres' and Paasche's index numbers? Prove that the Marshall-Edgeworth index number lies between Laspeyres' and Paasche's index numbers.  
(b) Discuss some uses of price index numbers. (3+5)+6
12. (a) Explain the term :  $3\sigma$  – limits.  
(b) Give some examples of defects for which the C-chart is applicable.  
(c) How do you calculate control limits for a C-chart? 4+4+6
13. (a) Define 'Crude death rate' and 'Standardized death rate'.  
(b) Explain why STDR's are computed, instead of CDRs, to compare the mortality situations of two different communities.  
(c) What do you mean by 'Cost of living index number' and 'Consumers price index number'? 4+6+4
14. Answer **any two** questions : 7×2  
(a) Write short note on Rational subgroup.  
(b) Explain the usefulness of R-chart.  
(c) Describe the different components of a complete life table.
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