2021

STATISTICS — HONOURS

Paper : CC-5

(Linear Algebra)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***any five*** questions : 2×5

(a) Find the value of the determinant in terms of cube root of unity ω

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

(b) For an idempotent matrix A of order n, show that Rank(A) = trace(A).

(c) For an elementary matrix E and a non-singular matrix A, show that $|EA| = |E|\,|A|$.

(d) Let $V = \mathbb{R}^n$ with $n = 3$, $\mathbb{F} = \mathbb{R}$, then show that $W = \{(x_1, x_2, x_3)' : \sqrt{2}x_1 = \sqrt{3}x_2\}$ is a subspace of V on $\mathbb{F}$.

(e) What is Caley Hamilton theorem?

(f) Express A^3 in terms of A^2, A and I_2 where $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$.

(g) For any square matrix A, prove or disprove Rank(A^2) + Rank(A^4) = 2Rank(A^3).

(h) Let λ be an eigenvalue of a non-singular matrix A, show that $\frac{1}{\lambda}$ is an eigenvalue of A^{-1}.

2. Answer ***any two*** questions : 5×2

(a) State and prove a necessary and sufficient condition for a square matrix to be idempotent.

(b) Let $V = \mathbb{R}^3$ with $n = 3$, $\mathbb{F} = \mathbb{R}$, $\underline{x}_1 = (-2, 4, 3)'$ and $\underline{x}_2 = (1, 3, 2)'$. Show that the vector sub space spanned by $\{\underline{x}_1, \underline{x}_2\}$ is

$[\{\underline{x}_1, \underline{x}_2\}] = \{(\alpha, \beta, -(\alpha+7\beta)/10) : \alpha, \beta \in \pi\}$

(c) For any two matrices A and B of orders $m \times n$ and $n \times p$, show that Rank (AB) $\leq$ min (Rank (A), Rank (B)). Hence show that rank of a matrix does not change by pre or post multiplication by a triangualr matrix with positive diagonal elements.

3. Answer ***any three*** questions : 10×3

(a) (i) If A and B are two non-singular matrices possibly of different orders, then $(A + CBD)$ is invertible if and only if $(B^{-1} + DA^{-1}C)$. is non-singular and in that case

$$(A + CBD)^{-1} = A^{-1} - A^{-1}\, C\, (B^{-1} + DA^{-1}C)\, DA^{-1}$$

(ii) Use the above result to find the inverse of the matrix

$$M = \begin{pmatrix} \alpha_1^2 + 1 & \alpha_1\alpha_2 & \alpha_1\alpha_3 \ldots \alpha_1\alpha_n \\ \alpha_2\alpha_1 & \alpha_2^2 + 1 & \alpha_2\alpha_3 \ldots \alpha_2\alpha_n \\ \vdots & \vdots & \vdots \quad \vdots \\ \alpha_n\alpha_1 & \alpha_n\alpha_2 & \alpha_n\alpha_3 \ldots \alpha_n^2 + 1 \end{pmatrix}.$$

(b) If A and B are non-negative definite (*n.n.d.*), then show that diag (A, B) is *n.n.d.* If A is positive definite and B is negative definite, what can be said about diag (A, B)?

(c) (i) Show that any square matrix A is of full rank iff $|A| \neq 0$.

(ii) Show that every skew symmetric matrix of odd order is singular.

(d) Find row space $\mathscr{R}(A)$ and column space $\mathscr{C}(A)$ of the matrix A where

$$A = \begin{bmatrix} 4 & 1 & 6 & 0 \\ 0 & 1 & 2 & -4 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Also find a basis of $\mathscr{R}(A)$ and a basis of $\mathscr{C}(A)$.

(e) (i) Find eigenvalues of a idempotent matrix A and show that $|A| \neq 0$ implies $A = I$.

(ii) Show that all eigenvalues of a square matrix A are non-zero if and only if A is non-singular.