# 2021

## STATISTICS — HONOURS

### First Paper

### (Group - B)

### Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

*Notations and symbols are as usual.*

Answer ***any four*** from ***question nos.*** **1-8** and ***any two*** from ***question nos.*** **9-12**.

**1.** State and prove the Markov's inequality. 5

**2.** When are two events $A$ and $B$ said to be independent? If $0 < P(B) < 1$, show that $A$ and $B$ are independent if and only if $P(A \mid B^C) = P(A)$. 5

**3.** A fair coin is tossed 10 times. Find the probability that two heads do not occur consecutively. 5

**4.** Define expectation of a random variable. Give an example of a random variable $X$ for which $0 < E(X) = E(X^2) < 1$. 5

**5.** Let $X$ and $Y$ be two independent random variables with $E(X) = E(Y) = 0$ and $\text{Var}(X) = \text{Var}(Y) = 1$. Show that for any $\lambda > 0$,

$$P(X^2 + Y^2 \geq 2\lambda) \leq \frac{1}{\lambda}\cdot$$ 5

**6.** What does correlation coefficient ($\rho$) measure? Interpret the situations for $\rho = 0$ and $\rho = \pm 1$. 5

**7.** An elevator starts with 6 persons and stops at 8 floors of a building. What is the probability that no two persons get down at the same floor? 5

**8.** Let $X$ and $Y$ be jointly distributed with variances $\sigma_x{}^2$ and $\sigma_y{}^2$ and correlation coefficient $\rho$. If $X$ and $Y - \rho\frac{\sigma_Y}{\sigma_X}X$ are independent, show that regression of $Y$ on $X$ is linear. 5

**9.** (a) Define a random variable with an illustrative example.

(b) Define the cumulative distribution function of a random variable and prove its any two properties.

(c) Give the axiomatic definition of probability. Hence obtain the classical definition as a special case. 5+5+5

**Please Turn Over**

**10.** (a) Distinguish between discrete and continuous random variables. Define a probability density function. Suggest, giving justifications, a probability density function '$f$' of a random variable $X$ where $f(x) = 5$ for at least one $x$. Find skewness ($\beta_1$– coefficient) and Kurtosis ($\beta_2$–coefficient) of $X$.

(b) Obtain any coefficient of skewness of a distribution with density

$$f(x) = \begin{cases} \dfrac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}, & 0 < x < 1 \\ 0 & , \text{ otherwise} \end{cases}$$

with parameters $\alpha, \beta > 0$. 10+5

**11.** (a) State and prove Bayes' theorem.

(b) An urn contains six red and four white balls. Two balls are drawn without replacement. What is the probability that the second ball is red, if it is known that the first is red? (2+6)+7

**12.** (a) If the random variables $X$ and $Y$ are such that $P(X = +1) = p$ and $P(X = 0) = 1 - p$, $0 < p < 1$; $P(Y = +1) = q$ and $P(Y = 0) = 1 - q$, $0 < q < 1$, and $E(XY) = pq$, then show that $X$ and $Y$ must be independently distributed. Are $X^2$ and $Y^2$ also independently distributed?

(b) Suppose $X$ and $Y$ are random variables with the joint density

$$f(x, y) = \begin{cases} C & \text{if } x, y > 0, 2x + y < 2 \\ 0 & \text{otherwise} \end{cases}.$$

Obtain, for $0 < y < 2$, the conditional distribution of $X$ given $(Y = y)$ and $E(X \mid Y = y)$. Hence derive the linear regression of $Y$ on $X$. (6+4)+(2½+2½)

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