## M. Sc. (Physics) 3rd Semester Examination 2021 PHY 513 (Nuclear and Particle Physics)

Answer in your own words as far as practicable. The marks on the right-hand side denote the full marks for the respective question.

Full marks: 50
Time: 2.5 hrs
(2 hours for answering and 30 minutes for downloading, scanning, and mailing back)

## Important instructions. Please read carefully.

(i) Please write your Examination Roll Number and Registration Number (from an earlier admit card) at the top of your answer script. They should be clearly legible. Do not write your name or class roll number anywhere.
(ii) Scan the complete answer script into a single pdffile. Mail it back to the e-mail address from where you got the paper, and nowhere else.
(iii) The answer script file must be named as CCXXXXXXPHYAAA.pdf, where CC is the College identifier, XXXXXX is the last six digits of university roll number and AAA is the paper code (for this paper, 513).
If you name it in any other way, your answer script may not be evaluated at all.
Example:

- For a CU student with roll number C91/PHS/201099, the answer script for PHY513 must be named CU201099PHY513.pdf
- For a student of Lady Brabourne College with roll number 031/PHS/201099, the answer script for PHY513 must be named LBC201099PHY513.pdf
- For a student of Gurudas College with roll number $313 / \mathrm{PHS} / 201099$, the answer script for PHY513 must be named GC201099PHY513.pdf
- For a student of Vivekananda College with roll number 544/PHS/201099, the answer script for PHY513 must be named VC201099PHY513.pdf

Answer any five questions
$5 \times 10$
Symbols, wherever used, have their usual meanings, unless specified otherwise explicitly.

1. (a) Find out the Coulomb energy of a nucleus ${ }_{Z}^{A} X$ assuming that the charge density is constant.
The binding energy velues of the two mirror nuclei ${ }^{23} \mathrm{Na}$ and ${ }^{23} \mathrm{Mg}$ are 186.56 MeV and 181.73 MeV respectively. Estimate the radius of a nucleus with $A=23$. You may use the relation $e^{2} /\left(4 \pi \epsilon_{0}\right)=1.44 \mathrm{MeV}$-fm.
(b) Find the spin-parity and the magnetic moment of the ground state of the nucleus ${ }^{43} \mathrm{Ca}$ from the extreme single particle model.
$(2+3)+(3+2)$
2. (a) Calculate the Gamow peak energy for the reaction ${ }^{4} \mathrm{He}+{ }^{12} \mathrm{C}$ at $\mathrm{T}=10^{8} \mathrm{~K}$ Deduce the formula that you use.
(b) Obtain the quantum mechanical expressions for scattering and reaction cross sections. Show that there can be scattering without reaction but the opposite is not true. $4+(4+2)$
3. (a) Explain why fission of heavy elements as well as fusion of light elements can generate energy.
(b) The energy values (in keV) and the spin-parity of low-lying levels of ${ }_{92}^{234} \mathrm{U}$ are as follows:
$0.0\left(0^{+}\right), 43.4\left(2^{+}\right), 143.0\left(4^{+}\right), 297.0\left(6^{+}\right), 499.0\left(8^{+}\right)$
What can you say about the shape of the nucleus from the level energies and the fact that odd-spin terms are absent in the above scheme?
(c) Two particles are placed in the level $j=7 / 2$ in a nucleus. Find out the allowed spinparity values of the combination if (i) both the particles are neutrons; (ii) one is a neutron and the other is a proton.
Estimate the energy level splitting due to a spin orbit potential given by $-f(r) \vec{l} \cdot \vec{s}$.
$3+3+2+2$
4. (a) Find the eigenvalues of the $P_{\sigma}=\left(1+\overrightarrow{\sigma_{1}} \cdot \overrightarrow{\sigma_{2}}\right) / 2$ for two nucleon wave functions with total spin values given by 0 and $1 \hbar$. The suffices 1 and 2 of the Pauli spin matrices refer to the two particles.
(b) Calculate the magnetic moment of a n-p system in ${ }^{1} P_{1}$ state.
(c) Obtain an expression for coherent scattering cross section for neutron scattering by parahydrogen.
$2+4+4$
5. (a) Explain why, in Fermi's theory of beta decay, we can neglect (as a first approximation) the momentum dependence of the leptonic wavefunctions.
(b) Explain whether the following transition is allowed or first forbidden, and Fermi, Gamow-Teller or mixed: $\frac{3}{2}^{+} \rightarrow \frac{1}{2}^{-}$.
(c) Given that parity is represented by the matrix $\gamma^{0}$, show how $\bar{\psi} \gamma^{\mu} \psi$ transforms under parity.
(d) Write the four-fermion point interaction Lagrangian for beta decay and related processes, giving both the terms. Which term gives rise to electron capture and which term gives rise to the process $n+e^{+} \rightarrow p+\bar{\nu}_{e}$ ? $2+2+3+(1+1+1)$
6. (a) If spacetime had two instead of four dimensions, what would be the mass dimension of the Lagrangian? Hence determine the mass dimensions of the Dirac field and the photon field in two spacetime dimensions.
(b) Write down the isotriplet and isosinglet two-nucleon states. Hence show that

$$
\frac{\sigma\left(\pi^{0} d \rightarrow p n\right)}{\sigma\left(\pi^{+} d \rightarrow p p\right)}=2
$$

with $\sigma$ denoting scattering cross-section.
(c) Consider the fundamental representation of $\mathrm{SU}(2)$ given by $U=e^{i \theta_{a} \tau_{a} / 2}$ where $\tau_{a}$ are the Pauli matrices. Show that $U$ and $U^{*}$ are equivalent representations. $3+(2+2)+3$
7. (a) Write down the diagonal Gell-Mann matrices.
(b) Put the antiquarks $\bar{u}, \bar{d}$ and $\bar{s}$ in the $I_{3}-Y$ diagram.
(c) Write the irreducible representations for $\mathbf{3} \times \mathbf{3}$ where $\mathbf{3}$ is the fundamental representation of $S U(3)$. Hence explain why there cannot be a two-quark hadron.
(d) Consider the process $A \rightarrow B+C$, where $A$ has spin 1 and $B$ has spin $\frac{1}{2}$. State, with reason, the possible value(s) of spin of $C$.
(e) Why the following processes are not allowed: (i) $n \rightarrow \pi^{+} \pi^{-}$, (ii) $\Lambda^{0} \rightarrow n+\gamma$ ?

