2022

PHYSICS

Paper : CBCC-B (PHS-GE-32)

(Material Physics)

Full Marks : 50

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Answer *any five* questions.

1. Consider the equation $\dfrac{dx}{dt} = rx - x^3$

   (a) When $r < 0$, sketch the phase portrait $\left(\dfrac{dx}{dt} \text{ vs. } x\right)$ and denote the fixed point. Explain the stability of the fixed point graphically. Close to the fixed point determine how $x(t)$ varies with t.

   (b) Considering $r = 0$ sketch the phase portrait and determine how $x(t)$ varies with t. Sketch $x(t)$ vs. t for both $r = 0$ and $r < 0$.

   (c) Sketch the phase portrait for $r > 0$ and denote the fixed points. Explain their stability graphically from the phase portrait. $(1+1+1)+(2+1)+(2+2)$

2. (a) Find all the fixed points (say, x^*) of $\dfrac{dx}{dt} = x^2 - x - \dfrac{6}{25}$.

   (b) Determine the rate of growth (or, decay) of $\eta (\equiv x - x^*)$ close to the fixed points of (a).

   (c) Using linear stability analysis determine the stability of the fixed points of $\dfrac{dx}{dt} = \sin x$. $2+3+5$

3. (a) Explain the origin of degeneracy in three-dimensional harmonic oscillator. Explain whether the energy eigenstates are the eigenstates of Parity operators or not. Consider a model system of 10 non-interacting fermions in a single three-dimensional isotropic harmonic oscillator of angular frequency ω. What is the ground state energy of this system of 10 fermions? The degeneracy of n-th state is $\dfrac{1}{2}(n+1)(n+2)$.

   (b) Write down the expressions for the electron and hole concentrations for an intrinsic semiconductor and hence extract the formula for the band gap of a semiconductor.

   (c) Explain transcritical bifurcation with an example. $(2+1+2)+3+2$

Please Turn Over

4. (a) Distinguish between top-down and bottom-up approaches in synthesis of nanomaterials. Which of the groups do the following synthesis process belong to— (i) Sol-gel and (ii) mechanical grinding?

(b) What is a quantum dot? How does the density of states of a quantum dot vary with energy?

(c) Write down two important features of 2d materials. How are the electrons in graphene different from that in metal?

$(3+1)+(1+1)+(2+2)$

5. (a) Consider N non-interacting electrons with quadratic dispersion relation $(E \propto p^2)$ in a system of size $L \times L$. Compute the Fermi energy E_F of the system at absolute zero temperature. Hence, compute the average ground state energy of the system in terms of E_F.

(b) Indicate which of the following wave functions is acceptable. If it is acceptable, then determine whether it is a ground state or an excited state.

(i) $\psi_1(x) = A(x^2 - 1)e^{-x^2} \ (-\infty \le x \le \infty)$

(ii) $\psi_2(\phi) = Be^{i\phi/4} \ (0 \le \phi \le 2\pi)$

(iii) $\psi_3(x) = C \coth(x)(-\infty \le x \le \infty)$

(A, B and C are constants)

(c) Consider a particle moving in a potential $V(x) = \lambda x^4$. How does the energy eigenvalue depend on the quantum number n and λ? What is the degeneracy associated with 9th energy eigenstate?

$(2+2)+3+(2+1)$

6. What is a magneto-resistance? Define Hall effect. Starting from the Drude equation find expressions for the magneto-resistance and Hall coefficient under the influence of transverse magnetic field.

10

7. (a) What are the difficulties of measuring very low conductivity material? Describe with appropriate circuit diagram a process for the measurement of resistance of such a material.

(b) During measurement of high resistance by the loss of charge method a 10F is charged to 30V and then discharged through an unknown resistance R. After one second the voltage across the resistance was found to be 15V. Calculate the value of R. [Use In2 = 0.69]

7+3