16/1/19

2018

PHYSICS

Paper : PHY : 412

(Classical Mechanics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any five*** questions.

1. (a) Two equal point-masses m, placed at $x = \pm a$ are connected by identical springs of spring constant k to another point mass M at $x = 0$. For small oscillation along X axis, find the frequencies of the normal modes of vibration.

   (b) A particle of mass m is released from rest at (0, 0) under uniform gravitational field. It has to reach a fixed point (x_0, y_0) in the least time. Show that the curve the particle has to follow is a cycloid. 5+5

2. (a) Show that if the Lagrangian of a system has no explicit time-dependence, the Hamiltonian is conserved.

   (b) Define Poisson bracket. Prove that an observable which is not explicitly dependent on time, will be conserved when its Poisson bracket with the Hamiltonian vanishes.

   (c) Show that the transformation

   $$Q = p + iaq, \quad P = \frac{p - iaq}{2ia}, \text{ with } i^2 = -1 \text{ and } a = \text{constant},$$

   is canonical. Also find the generating function of type 2 for it. 2+3+5

3. (a) From the Hamiltonian for Simple Harmonic Oscillator $H = \frac{p^2}{2m} + \frac{1}{2}kq^2$

   Obtain the solution $Q = \sqrt{\frac{2E}{m\omega^2}} \sin \omega(t + \beta)$ by Hamilton-Jacobi method.

   (b) A particle slides down a cycloidal track $x = l(2\phi + \sin 2\phi)$, $y = l(1 - \cos 2\phi)$ in a vertical plane without friction under gravity. Using action-angle variables, find the frequency of oscillation for $\phi \leq \pi/2$. 5+5

4. (a) Find the fixed points of the map $x_{n+1} = ax_n - bx_n^2$ where a, b are constant parameters and $0 \le x_n \le 1$. Identify the range of values of the parameters for which the fixed points are stable.

(b) Define streamline and vortex line for a fluid. What are the conditions for a fluid to satisfy Bernoulli's theorem?

(c) Consider the two-dimensional flow of an incompressible liquid. If the flow is irrotational, one can write the fluid velocity $\vec{q} = -\nabla\phi$. Show that in a steady state, ϕ cannot have any local minimum or maximum in the plane of flow of the liquid. 5+2+3

5. (a) Find the number of degrees of freedom of an n-dimensional rigid body moving in n dimension.

(b) What is an orthogonal transformation? Show that any symmetric tensor of rank two can be diagonalized by an orthogonal transformation.

(c) The Lagrangian of a rotating symmetric top of mass M is given by (symbols having their usual meanings)

$$L = \frac{1}{2}I_1\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) + \frac{1}{2}I_3\left(\dot{\phi}\cos\theta + \dot{\psi}\right)^2 - Mgh\cos\theta.$$

Find three constants of motion. Show how one can reduce the energy equation to a single-variable cubic equation. Argue that if the top rotates, there must be two and only two possible roots of that equation in the physically allowed region for θ. 2+(1+2)+(2+2+1)

6. (a) Write down the wave equation in covariant form. What is four divergence?

(b) Starting from the relativistic action of a free particle, get the Lagrangian of the same.

(c) An electron of rest mass 0.51 MeV/c^2 and a proton of rest mass 0.938 GeV/c^2 have energies of 1 GeV each. Find the time taken for each of them to travel a distance of 10 m.

(d) A particle is moving with a velocity given by $v/c = (e-1)/(e+1)$ where e is the base of natural logarithm. Find its rapidity. Deduce the expression that you use. 2+2+3+3

7. (a) A high energy proton strikes another proton at rest and creates a proton-antiproton pair in addition to the original particles. Find the threshold energy (minimum energy) of the incident proton for the reaction to occur.

(b) Consider the scattering $A + B \rightarrow C + D$. Define the Mandelstam variables in terms of the four-momenta, and show that $s + t + u = (m_A^2 + m_B^2 + m_C^2 + m_D^2)\,c^2$ (you can use the natural system with $c = 1$).

(c) The KEK-B collider hit 3.5 GeV positrons with 8 GeV electrons. Deduce the value of $\sqrt{s}$. This produced the Υ meson, whose mass is 10.5794 GeV/c^2. What is its velocity in the lab frame?

4+3+(2+1)

(b) Determine the canonical transformation $(q, p) \rightarrow (Q, P)$ for the following generating function :

$$F(Q, p) = -(e^Q - 1)^2 \tan(p).$$

(c) Show that the following transformation is canonical :

$$q = P^2 + Q^2 \text{ and } p = \frac{1}{2}\tan^{-1}(P/Q).$$

(1+3)+3+3

7. (a) Suppose a particle of mass m is moving in an inverse square central force field $V(r) = -K/r$ (r : generalised coordinate). The conjugate momenta corresponding to radial and angular components are p_r and p_θ. Apply Hamilton-Jacobi method and show that the equation of motion is a conic section.

(b) Determine the frequency of a harmonic oscillator of mass m, force constant k by the method of action angle variables. Using it, obtain the expressions of old generalised coordinate and momentum in terms of canonically transformed generalised coordinate and momentum.

(c) A particle is thrown vertically upward with an initial velocity u against the gravity. Apply Hamilton-Jacobi method and determine the general solution of equation of motion. 5+3+2