

**GURUDAS COLLEGE**  
**B.SC. PART I (HONOURS) EXAM - 2020**  
**FIRST PAPER**  
**SUBJECT - MATHEMATICS**

*Time Allotted : 2 Hours*

*Full Marks : 50*

\*\*\*\*\*  
**Instruction: The scanned copy of the answer script should be submitted through the email - [gcmath.online.exam@gmail.com](mailto:gcmath.online.exam@gmail.com) with in 30 minutes after the end of Examination to the Mathematics Department.**  
\*\*\*\*\*

**GROUP - A (Module - I)**  
**Marks - 25**

Answer any **five** questions :

1. Reduce the equation  $x^3 - 3x^2 + 12x + 16 = 0$  to its standard form and then solve the equation by Cardan's method. 5
2. If  $a, b, c$  are positive real numbers prove that  $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} > \frac{3}{2}$ , unless  $a = b = c$ . 5
3. Prove that the equation  $(x + 1)^4 = a(x^4 + 1)$  is a reciprocal equation if  $a \neq 1$  and solve it when  $a = -2$ . 5
4. If  $x = \cos \alpha + i \sin \alpha$ ,  $y = \cos \beta + i \sin \beta$ ,  $z = \cos \gamma + i \sin \gamma$  and  $x + y + z = xyz$ , prove that  $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -1$ . 5
5. How many integers are exist in the interval  $(1, 1500)$  which satisfy  $x \equiv 2 \pmod{5}$ ,  $x \equiv 3 \pmod{7}$ ,  $x \equiv 4 \pmod{11}$  ? 5
6. (a) Find all elements of order 8 in the group  $(\mathbb{Z}_{24}, +)$ , where  $(\mathbb{Z}_{24}, +)$  represents the additive group of residue classes of integers modulo 24. 2  
(b) Let  $(G, *)$  be a group and  $a \in G$ . Let  $H = \{x \in G : x * a = a * x\}$ . Prove that  $(H, *)$  is a subgroup of  $(G, *)$ . 3

**GROUP - B (Module - II)**  
**Marks - 25**

Answer any **five** questions :

7. Show that the quantity  $ab - h^2$  of the quadratic expression  $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$  remains invariant upon rotation of coordinate axes through an angle  $\theta$ . 5
8. Show that the acute angle between the pair of straight lines  $ax^2 + 2hxy + by^2 = 0$ ,  $h^2 - ab > 0$  is  $\tan^{-1} \frac{2\sqrt{(h^2 - ab)}}{(a+b)}$ . 5

9. Show that the general equation of second degree  $x^2 - 5xy + 6y^2 + 11x - 17y + 13 = 0$  represents an ellipse and find its canonical form. 5
10. Find the polar equation of a straight line at a distance  $p$  from the pole and perpendicular to the initial line. 5
11. Find the necessary and sufficient condition that two non-parallel straight lines  $(x - x_i)/l_i = (y - y_i)/m_i = (z - z_i)/n_i$ ,  $i = 1, 2$  to be coplanar. 5
12. Find the radius of the circle  $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ ,  $x + 2y + 2z = 15$ . 5