2022

PHYSICS

Paper : PHY 523

(Nonlinear Dynamics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any five*** questions.

1. (a) Make a schematic plot of the potential $V(x)$ for the one-dimensional system $\dot{x} = x - x^3$. Identify all the equilibrium point(s). Plot the phase trajectories close to the equilibrium point(s).

   (b) Show that the initial value problem $\dot{x} = x^{1/3}$ with $x(0) = 0$ does not have a unique solution. What happens if the initial value is $x(0) = x_0 \neq 0$? (2+2+2)+(3+1)

2. (a) Consider the system given by $\dot{x} = r + x^2$. Show that the system undergoes a bifurcation as the parameter r is varied from $r > 0$, $r = 0$ and $r < 0$. Classify the nature of the fixed points for $r < 0$ and discuss what happens when $r \to 0$. Draw the bifurcation diagram.

   (b) Consider the system $\dot{x} = rx + x^3 - x^5$.

   (i) Obtain algebraic expressions for all the fixed points as the prarameter r takes different values from negative to positive, through zero.

   (ii) For different r values, sketch the vector fields. Clearly indicate all the fixed points and their stability. (1+1+1)+(4+3)

3. (a) Enumerate the important features of a limit cycle. Discuss how it differs from the regular closed orbits which may occur in a linear system.

   (b) Consider the system $\dot{r} = r(1 - r^2) + \mu r \cos\theta, \dot{\theta} = 1$. Identify a trapping region and hence, using Poincare- Bendixon theorem, show that a limit cycle exists for positive but sufficiently small μ. (3+2)+5

4. (a) Consider the dynamics of a point (x, y) described by the equations

$$\frac{dx}{dt} = -y \quad \frac{dy}{dt} = x + ay,$$

   where a is a real parameter. Find the stability of the fixed point for $0 < a < 2$.

(b) The equation of motion of a pendulum, in the absence of damping and external driving, in dimensionless units is given by,

$$\ddot{\theta} + \sin(\theta) = 0 .$$

Linearize the system and find the Jacobian matrix at all the fixed points. Argue why the fixed point at the origin is a nonlinear centre. 5+(3+2)

5. (a) Consider the map $x_{n+1} = rx_n - x_n^3$ for real values of x. Locate the fixed points and find the range of values of r for which the map has one or more stable fixed point(s). Find the values of r for which the fixed points are (i) superstable (ii) marginally stable (iii) unstable.

(b) For the map $f(x, a) = -(1 + a)x + x^2$, define the universal constants δ. Given that the renormalisation equation for the bifurcation point a_k is $a_k = a_{k+1}^2 + 4a_{k+1} - 2$, find the vlaue of δ. 5+5

6. (a) Define the similarity dimension of a self-similar fractal set and find the dimension of Koch curve. What is the length of Koch curve?

(b) Consider the Henon map,

$$x_{n+1} = 2x_n^2 + 2Cx_n - y_n$$
$$y_{n+1} = x_n$$

where C is a parameter. (i) Show that this map is area-preserving. (ii) Find the (1-cycle) fixed points. (iii) Find the regions of values of C for which the fixed points are stable. 4+(1+1+4)

7. (a) Consider the Lorenz equations

$$\dot{X} = -\sigma X + \sigma Y$$
$$\dot{Y} = -XZ + rX - Y$$
$$\dot{Z} = XY - bZ.$$

in usual notation. Show that the origin is a fixed point and locate the other fixed points. Show that the origin is stable for $r < 1$ and unstable for $r > 1$.

(b) For the map $x_{n+1} = \sqrt{x_n}$ with positive values of x, find the fixed point(s) and analyse their stability. Will there be any two-cycle fixed point?

(c) What are the basic characteristics of a strange attractor? 4+4+2

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