Gurudas College

M. Sc.(Physics) 2nd Semester Internal Examination 2021

Paper(Subject): PHY 422 (Quantum Mechanics II)

Time: 1Hr Full Marks:25

Answer any five questions from below.

1. (a) Starting from the Schroedinger's equation for a projectile particle of mass m and initial wave vector $\vec{k}$ being scattered by a central potential $V(r)$, obtain the Lippmann-Schwinger equation.
   (b) Starting from the Lippmann-Schwinger equation how is the Born series constructed?
   c) State the optical theorem.
   2+2+1

2. For a particle in a one dimensional potential $V(x)$ show that the solution of the Schrodinger equation in the WKB approximation is given as $\psi(x) = A\frac{1}{\sqrt{k(x)}}exp(\pm\int^{x} k(x')dx')$ Where $k(x') = \frac{\sqrt{2m(E-V(x))}}{\hbar}$ is the local wave vector.
   Sketch the potential $V(r) = V_0 ln(\frac{r}{a})$ (V_0 and a are Constants) . Show the turning points in the graph. 4+0.5+0.5

3. Consider a one-dimensional simple harmonic oscillator with angular frequency ω. For $t < 0$, it was known to be in the ground state, and for $t \geq 0$ it is subjected to a time-dependent perturbation $\hat{H}'(t) = A\hat{x}e^{-t/\tau}$ where A, τ are constants in both space and time. Using time-dependent perturbation theory to first order, calculate the probability of finding the oscillator in its first excited state at $t > 0$. Justify your result in the limit $t \to \infty$.

   4+1

4. (a) Suppose a spinless particle with angular momentum $\vec{L}$ is bound to a fixed centre by a potential $V(\vec{r})$ so asymmetrical that no energy level is degenerate. Using time reversal invariance, prove $<\hat{\vec{L}}>= 0$ for any energy state.

   (b) Consider the 2×2 matrix defined by

$$U = \frac{a_0 + i\vec{\sigma}.\vec{a}}{a_0 - i\vec{\sigma}.\vec{a}}$$

where a_0 is a real number and $\vec{a}$ with real components. Prove that U is unitary and unimodular.

(c) Consider the scalar product of two vector operators $\vec{A}$ and $\vec{B}$. Verify that $\vec{A}.\vec{B}$ is a scalar operator. (2+2+1)

5. (a) Consider a system with $j = 1$. Explicitly write

$$< j = 1, m^{'} | \hat{J}_y | j = 1, m >$$

in 3×3 matrix form.

(b) Show that for $j = 1$ only, it is legitimate to replace $e^{-i\frac{i}{\hbar}\hat{J}_y\beta}$ by

$$I - i(\frac{\hat{J}_y}{\hbar})sin\beta - (\frac{\hat{J}_y}{\hbar})^2(1 - cos\beta)$$

. (2+3)

6. a) Show that $\bar{\psi}\gamma_\mu\gamma_5\psi$ is a vector under Lorentz transforamation and a pseudo/axial vector under parity transformation. Where ψ satifies Dirac equation.
b) Show that $\mathrm{Tr}(\gamma_\mu\gamma_\alpha\gamma_\sigma) = 0$. (1.5+1.5)+2

7. Show if γ_μ' s satisfy the anti-commutation relation $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, similarity transformed $\gamma'_\mu = S^{-1}\gamma_\mu S$ will also satisfy same anti-commutation relation.
b) Construct Dirac four current. Show that probablity density: the 0-th component of Dirac four current is positive definite. 2+(2+1)

8. Helicity operator defined in 4×4 matrix space $h = \frac{\hbar\vec{\Sigma}.\vec{p}}{2|\vec{p}|}$. Where $\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$
a) Show that h^2 has eigenvalue $\frac{3}{4}\hbar^2$.
b) Show that h commutes with free Dirac Hamiltonian. 2+3