2019

PHYSICS

Paper : PHY-512

(Solid State Physics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

(Symbols have their usual meaning.)

Answer ***any five*** questions.

1. (a) Iron crystallizes in the *bcc* structure at room temperature with the mass density given by $\rho = 7.86\ \text{g/cm}^3$. Compute the radius of an iron atom in the *bcc* structure.

   (b) Prove that the ideal c/a ratio for the hexagonal close packed structure is $\sqrt{8/3}$.

   (c) The time dependent electric field $\mathbf{E}(\mathbf{r}, t)$ and Polarization $\mathbf{P}(\mathbf{r}, t)$ are real quantities. Their relation is

$$P_{\alpha\beta}(\mathbf{r}, t) = \epsilon_0 \sum\nolimits_{\beta} \chi_{\alpha\beta}(\mathbf{r}, t) E_{\beta}(\mathbf{r}, t)$$

   where $\chi_{\alpha\beta}(\mathbf{r}, t)$ is a component of the electric susceptibility tensor.

   (i) Show that the complex conjugate of the wave vector and the frequency domain electric susceptibility is given by

$$\chi^*_{\alpha\beta}(\mathbf{q}, \omega) = \chi_{\alpha\beta}(-\mathbf{q}, -\omega)$$

   (ii) Determine the symmetry of the real and imaginary parts of $\chi^*_{\alpha\beta}(\mathbf{q}, \omega)$. 3+2+5

2. (a) Consider the Landau free energy for second order ferroelectric transition. Determine and plot the temperature dependence of the inverse susceptibility just above and below the critical temperature.

   (b) Obtain Bloch's equations in context with the nuclear magnetic resonance. Hence interpret the spin-spin relaxation time.

   (c) Define molecular polarizability and explain its different components. 4+4+2

3. (a) Two superconductors of the same material are separated by a thin tunnel barrier and a voltage V is applied between them. Write down the two coupled Schrödinger equations to describe the system and hence define AC Josephson effect. Show that the junction current oscillates with frequency $\omega = 2eV/\hbar$.

(b) The orbital part of the wave function of a Cooper pair is expanded in terms of the plane waves as

$$\phi(\mathbf{r}) = V^{-1/2}\sum_{\vec{k}}{}' a_{\vec{k}} e^{i\vec{k}\cdot\vec{r}}$$

with a_k's to be determined from

$$(2\epsilon_k - \varepsilon)a_k = V\sum{}'_{k'} a_{k'}$$

where $\varepsilon \approx 2\epsilon_F - \Delta$, where Δ is the energy gap.

Evaluate $\langle r^2 \rangle$ for a Cooper pair, where $\mathbf{r}$ is the inter-electron vector. Justify all the approximations made and express $\langle r^2 \rangle$ in terms of Δ at zero temperature and Fermi velocity v_F. (3+2)+5

4. (a) State Friedel's law. Derive an expression for the atomic scattering factor.

(b) Show that the sum $\sum_m e^{-i\Delta\mathbf{k}\cdot\mathbf{r}_m}$ will exist only when $\Delta\mathbf{k}$ is equal to the reciprocal lattice vector. Use this relation to explain the construction of the Ewald sphere.

(c) The constant volume heat capacity per atom, c_V, due to lattice vibration for copper is 0.38J/mol-K at 20K. If the Debye temperature is 340K, calculate c_V at 30K and 450K. Justify the formula you are using.

(d) State in which of the following materials the net magnetisation is zero giving reasons :

(i) ferromagnetic (ii) paramagnetic (iii) antiferromagnetic and (iv) ferrimagnetic. 2+3+3+2

5. (a) Consider the Hamiltonian H for N free electrons in a magnetic field h with

$$H = -h\sum_i S_i.$$

Show that the susceptibility is inversely proportional to the temperature T.

(b) When the electrons are in a crystal, derive the expression of the susceptibility.

(c) Consider an alloy of zinc and copper in a three dimensional lattice. Defining c to be the concentration of Zn and a the lattice parameter in the FCC lattice of the alloy, find out the electron density and magnitude of the Fermi wave vector k_F as a function of a and c.

Given that the primitive vectors of the reciprocal lattice in the FCC lattice are $\frac{2\pi}{a}(1, 1, -1)$, $\frac{2\pi}{a}(1, -1, 1)$ and $\frac{2\pi}{a}(-1, 1, 1)$, show that the crystal is expected to undergo a structural transition at $c \approx 0.36$.

3+3+(2+2

6. (a) Show that in the tight binding approximation, the energy expression for electrons in a solid is given by

$$E(k) = E_{at} + \alpha + \sum_{R \neq 0} A(\mathbf{R}) e^{i\mathbf{k}.\mathbf{R}}$$

where $A(\mathbf{R})$ is the overlap between atomic wave functions centered at $\mathbf{R} = 0$ and at $\mathbf{R}$: $\alpha = A(0)$ and E_{at} denotes the energy in an isolated atom. What are the conditions under which tight binding approximation is justified?

(b) Using tight binding approximation, find $E(k)$ and the bandwidth for a simple cubic lattice. Find also the velocity and the effective mass of the electrons at the first Brillouin zone boundary in the one dimensional case.

(3+1)+(3+1+2)

7. (a) Find the dispersion relation for a linear chain of atoms which have alternating spring constants C_1 and C_2.

(b) In a crystal with p atoms in the primitive cell, how many acoustic and optical branches of vibrations are there?

(c) Starting from the Boltzmann transport equation for electrons in an electric field $\vec{\varepsilon}$,

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla_{\vec{r}} f - \frac{e}{\hbar}\vec{\varepsilon}.\nabla_{\vec{k}} f = \left(\frac{\partial f}{\partial t}\right)_s$$

where the term on the r.h.s. represents scattering, and using the relaxation time ansatz, show that the non-equilibrium Fermi distribution function f is given by

$$f(\vec{k}) = f_0\left(\vec{k} + \frac{e\vec{\varepsilon}}{\hbar}\tau\right)$$

where f_0 is the equilibrium value of f and τ is the relaxation time.

4+2+4

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