2021

STATISTICS — HONOURS

Paper : DSE-B-2

(Stochastic Process and Queuing Theory)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Fill in the blanks (***any five***) [if you answer more than ***five***, only the first five will be checked.] : 2×5

(a) If $(X_n)n \geq 1$ are independent, $X_n \sim N\left(\frac{1}{2}, \frac{1}{2^2}\right)$ if n is odd and $X_n \sim$ Exp(2) if n is even where Exp(λ) stands for the exponential distribution with mean $1/\lambda$; then the sequence $(X_n)n \geq 1$ is ________ stationary.

(b) A state leading to an absorbent state in a Markov chain must itself be________.

(c) A finite state Markov chain cannot have any________state.

(d) If *a, b, c* and *d* have respective mean return times 3, 4, 5 and 6 in an irreducible Markov chain on states $\{a, b, c, d, e\}$, then the mean return time of e is ________.

(e) A pure birth process with equal birth rates is a ________ process.

(f) In the Kolmogorov backward equations in matrix form, $P'(t)$ equals________, where $P(t)$ is the transition probability matrix at time t.

(g) A queueing discipline where customers arriving most recently are served before those waiting from earlier is called ________.

(h) In a single-server Markovian queue with arrival rate 10 per hour and service rate 15 per hour, the limiting mean queue length is ________.

2. Write short notes ***fully in your own words*** on ***any two*** of the following : 5×2

(a) Stationary distribution for the Ehrenfest chain

(b) Equality of periods of communicating states

(c) The Yule-Furry process.

3. Write essays ***fully in your own words*** on ***any three*** of the following : 10×3

(a) Number of visits to a recurrent state

(b) Roles of positive recurrence and aperiodicity in ergodicity of an irreducible Markov chain

(c) Transience of asymmetric simple random walk

(d) Conditional distribution of arrival times of a Poisson process (N_t) up to time T given the value of N_T and an application

(e) Balance equations for a birth and death chain with interpretation.