Gurudas College (University of Calcutta)

M. Sc. Semester 2, Internal Examination, 2020

Subject: PHYSICS

Paper: PHY 423 (Statistical mechanics)

Time: 1hr Full marks: 25

Answer any five questions from below.

1. (a) What is Liouville theorem in classical statistical mechanics ? Discuss the phenomenon of relaxation in the light of this theorem.
   (b) Give an appropriate definition of chemical potential.
   (c) Explain whether a ^{3}He atom is a Boson or a Fermion. (1+2)+1+1

2. Discuss why all the three equilibrium ensembles give identical results in the thermodynamic limit in almost all cases. 5

3.(a) Derive how a density operator changes with time.
(b) In an ensemble for spin 1/2 $particles$, 50% $are\ in\ the\ up\ state\ of\ the\ operarator$ S_x and the rest are in the down state of the same operator. Obtain the density matrix of the ensemble in the basis consisting of the eigenstates of the operator S_z.

2+3

4.(a) Discuss whether the F-Ddistribution function is applicable for all collections of identical Fermions.
(b) Explain why the value of the chemical potential for photons is zero.
(c) Determine the highest possible value of the chemical potential of an ideal gas of identical Bosonic atoms trapped in a 3-D simple harmonic potential.

1+2+2

5. Consider the Ising ferromagnet with zero field with the partition function given as $Z = \prod_{i=1}^{N} \{\sum_{s_i} e^{\beta Jzms_i}\}$ where the spin can take three values $s_i = 0, \pm 1$. Find the partition function in a compact form. Also find the mean field free energy and magnetization.

1+2+2=5

6. a) Find the critical exponent for the following function :

$$f(t) = at^{-\frac{4}{5}}(t+b)$$

where $t = \frac{T-T_c}{T_c}$.

b) Consider a system with Landau free energy :

$$f(M) = a(T - T_C)M^2 + bM^4 + cM^6 - HM$$

If $c > 0$, evaluate β when $b = 0$. Remember, the critical exponent β is defined as $H = 0,\ t \to 0,\ M \sim |t|^\beta$. 2+3=5

7. a) Given the velocity of a Brownian particle $v(t) = v_0 e^{-\gamma t} + \frac{1}{m}\int_0^t dt_1 \eta(t_1) e^{-\gamma(t-t_1)}$, show that

$$\langle v(t)v(t')\rangle = \frac{k_B T}{m} e^{-\gamma(t-t')}$$

b) Find $\langle X^2(t)\rangle$ and discuss the limits $\gamma t \to 0$ and $\gamma t \gg 1$.

2+(1+1+1)=5