2022

MATHEMATICS — HONOURS

Paper : CC-10

(Mechanics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings unless otherwise stated.

1. Answer the following multiple choice questions with only one correct option. Choose the correct option with proper justification : (1+1)×10

(a) A system of coplanar forces has total moments H, $2H$ and $3H$ about the points (0, 0), (0, 1) and (2, 4) respectively. Then the equation of the line of action of the resultant is

(i) $x - 2y = 1$ (ii) $x - y = 1$

(iii) $x + 2y = 1$ (iv) $x + y = 1$.

(b) The condition for equilibrium of a particle constrained to rest on a rough curve under any given forces, with usual notations, is

(i) $\left(X - Y\frac{dy}{dx}\right)^2 \le \mu^2\left(X\frac{dy}{dx} - Y\right)^2$ (ii) $\left(X + Y\frac{dy}{dx}\right)^2 \le \mu^2\left(X\frac{dy}{dx} - Y\right)^2$

(iii) $\left(X - Y\frac{dy}{dx}\right)^2 \le \mu^2\left(X\frac{dy}{dx} + Y\right)^2$ (iv) $\left(X + Y\frac{dy}{dx}\right)^2 \le \mu^2\left(X\frac{dy}{dx} + Y\right)^2$.

(c) Which of the following forces do appear in the equation of virtual work?

(i) Forces of action and reaction between two particles when the distance between them remains invariable.

(ii) Weight of a body or of a system of bodies.

(iii) The reaction of a smooth surface when the body in contact with it slides on the surface due to displacement.

(iv) The reaction of a fixed surface when the body in contact with it rolls without sliding on the surface due to displacement.

(d) A uniform ladder rests in limiting equilibrium with its lower end on a rough horizontal plane and its upper end against a smooth vertical wall. If θ be the inclination of the ladder to the vertical and μ be the coefficient of friction, then

(i) $\tan\theta = \mu$ (ii) $\tan\theta = 2\mu$

(iii) $\cot\theta = \mu$ (iv) $\cot\theta = 2\mu$.

(e) For a rectilinear motion of a particle if an impulse I changes its velocity from u to v and E is the change of kinetic energy, then

(i) $E = I\left(\frac{2u+3v}{5}\right)$ (ii) $E = I\left(\frac{u+v}{2}\right)$

(iii) $E = I\left(\frac{u+2v}{3}\right)$ (iv) none of these.

(f) A particle moving along a straight line has the relation $v^2 = 4 - x^2$ between the velocity v and displacement x from origin O at any time t, then the motion is

(i) S.H.M. with period $\frac{\pi}{2}$ (ii) S.H.M. with period π

(iii) S.H.M. with period 2π (iv) S.H.M. with period $\frac{3\pi}{2}$.

(g) A smooth sphere falling directly from a height H impinges on a horizontal fixed plane and rebounds to a height h. If e is the coefficient of restitution between the sphere and fixed plane, then

(i) $h = eH^2$ (ii) $h = (-e^2H)$

(iii) $h = e^2H$ (iv) $h = eH$.

(h) A particle describes the curve $p^2 = ar$ under a force F to the pole. Then the law of force is

(i) F varies as r (ii) F varies as r^2

(iii) F varies as r^{-2} (iv) F varies as r^{-3}.

(i) Equation of the path in a central orbit is $r = a(1 + 2\sin\theta)$. Then the apsidal distances are equal to

(i) $2a$ and $a/2$ (ii) $3a$ and a (iii) $4a$ and $2a$ (iv) $5a$ and $3a$.

(j) A force $\vec{F} = 2\vec{i} - \vec{j} - 2\vec{k}$ acts on a particle located at $P(0, 3, -1)$. Then the moment of the force $\vec{F}$ about the axis through O in the direction of the vector $3\vec{i} - 4\vec{k}$ is

(i) $-\frac{3}{5}$ (ii) $\frac{5}{3}$

(iii) $\frac{3}{5}$ (iv) $-\frac{5}{3}$.

Unit – 1

(Marks : 05)

2. Answer ***any one*** question : 5×1

(a) A ladder, whose centre of gravity divides it into two portions of lengths a and b, rests with one end on a horizontal floor and the other end against a rough vertical wall. Show that the inclination of the ladder to the floor, when it is in limiting equilibrium, is $\tan^{-1}\left[\dfrac{a-b\mu\mu'}{\mu(a+b)}\right]$, where μ and μ' are the coefficients of friction of the floor and the wall respectively.

(b) Forces X, Y, Z act along the three lines given by the equations $y=0, z=1; z=0; x=1; x=0, y=1$. Prove that the pitch of the equivalent wrench is $\dfrac{YZ+ZX+XY}{X^2+Y^2+Z^2}$.

Unit – 2

(Marks : 05)

3. Answer ***any one*** question : 5×1

(a) Six equal rods each of weight w, are freely jointed at their ends to form a regular hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string. Prove that its tension is $3w$.

(b) A body consisting of a cone and a hemisphere on the base rests on a rough plane table, the hemisphere being in contact with the table. Show that for the stability of the equilibrium, the greatest height of the cone is $\dfrac{\sqrt{3}}{2}$ times the diameter of the hemisphere.

Unit – 3

(Marks : 10)

4. Answer ***any two*** questions : 5×2

(a) Deduce the tangential and normal components of velocity and acceleration of a particle moving in a plane.

(b) A particle describes a path with an acceleration μy^{-3} which is always parallel to the axis of y and directed towards the x-axis. If the particle is projected from a point $A(0, a)$ with velocity $\sqrt{\mu}/a$ parallel to x-axis, then show that the path described is a circle.

(c) An engine is pulling a train and works at a constant rate doing H units of work per second. If M be the mass of the whole train and R be the resistance, supposed constant, show that the time required in generating a velocity V from rest is $\dfrac{MH}{R^2}\log\left(\dfrac{H}{H-VR}\right)-\dfrac{MV}{R}$ seconds.

(d) A particle is projected from the lowest point with velocity 'u' and moves along the inside arc of a smooth vertical circle. Show that if $u^2 \geqslant 5ga$, then the particle reaches the highest point and makes a complete revolution of the circle.

Unit – 4

(Marks : 15)

5. Answer ***any one*** question :

(a) A particle moves from rest in a straight line under an attractive force $\mu \times (\text{distance})^{-2}$ per unit mass to a fixed point on the line. Show that if the initial distance from the centre of force be $2a$, then the distance will be a after a time $\left(\frac{\pi}{2}+1\right)\left(\frac{a^3}{\mu}\right)^{\frac{1}{2}}$. 7

(b) A particle is projected vertically upwards under gravity with a velocity V. Assuming that the resistance of air is 'kv' per unit mass, where v is the velocity of the particle at time t and k a constant, show that particle comes to rest at a height $\frac{V}{k}-\frac{g}{k^2}\log\left(1+\frac{kV}{g}\right)$ above the point of projection. If instead of being projected upwards, the particle falls downwards from rest, find the distance traversed by the particle in time t. 4+3

6. Answer ***any one*** question :

(a) A particle is projected with velocity $\sqrt{\frac{2\mu}{3a^3}}$ at right angles to the radius vector at a distance 'a' from a centre of attracting force $\frac{\mu}{r^4}$ per unit mass, r being the distance of the particle from the centre of force. Find the path of the particle and show that the time it takes to reach the centre of force is $\frac{3\pi}{8}\sqrt{\frac{3a^5}{2\mu}}$. 4+4

(b) A heavy particle slides down a rough cycloid whose base is horizontal and vertex downwards. Show that if ϕ be the angle made by the tangent to the path with the horizontal at any time t,

$$\frac{d^2}{dt^2}\left[e^{-\phi\tan\lambda}\sin(\phi-\lambda)\right]=-\frac{g}{4a}\sec^2\lambda\, e^{-\phi\tan\lambda}\sin(\phi-\lambda),$$

where λ is the angle of friction and 'a' the radius of the generating circle. If the particle starts from rest at a cusp, prove that it will come to rest before reaching the vertex if $e^{\frac{\pi}{2}\tan\lambda}>\cot\lambda$. 6+2

Unit – 5

(Marks : 10)

7. Answer ***any one*** question :

(a) (i) A gun of total mass M tons, free to recoil horizontally, fires a shot of mass m tons. If the gun is fired with the barrel inclined at an angle α to the horizontal, prove that the shot is actually projected at an angle $\tan^{-1}\left[\left(1+\dfrac{m}{M}\right)\tan\alpha\right]$ to the horizontal.

(ii) An elastic ball of mass 'm' falls from a height 'h' on a fixed horizontal plane and rebounds. If the coefficient of restitution be 'e', show that the loss of K.E. by the impact is $mgh\,(1-e^2)$.

6+4

(b) (i) State the principle of conservation of linear momentum for a many particle system.

(ii) What do you mean by degrees of freedom of a mechanical system? What is the number of degrees of freedom of a simple pendulum moving in a vertical plane?

(iii) If a rocket, originally of mass M, ejects a mass $\dfrac{M}{n}$ per unit time with relative velocity V and if $\dfrac{M}{k}$ is the mass of the case and allied objects in the rocket, show that it can not rise vertically at once unless $V > ng$.

(iv) Reduce the two-body problem into a one-body problem. 2+(1+1)+3+3

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