2021

MATHEMATICS — HONOURS

Fourth Paper

(Module – VIII)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

(Marks : 15)

1. Answer ***any one*** of the following :

(a) Find the equation of the cylinder whose generators are parallel to the straight line $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=9,\ z=1.$ 3

(b) Find the cartesian coordinates of the point whose spherical coordinates are $\left(2, -\pi/6, \pi/4\right)$. 3

2. Answer ***any two*** of the following : 6×2

(a) Show that only one tangent plane can be drawn to the sphere $x^2+y^2+z^2-2x+6y+2z+8=0$ through the straight line $3x - 4y - 8 = 0 = y - 3z + 2$.

(b) The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ by the plane $z = 0$ is a parabola. Show that the locus of the vertex of the cone is a pair of planes $z = \pm c$.

(c) Prove that the plane $ax + by + cz = 0$ $(abc \neq 0)$ cuts the cone $yz + xz + xy = 0$ in perpendicular straight lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0.$

(d) Find the equations of the generators of the paraboloid $4x^2-y^2=4z$ passing through the point $(-3, 2, 8)$.

Group – B

(Marks : 10)

3. Answer ***any one*** of the following :

(a) (i) A square frame $ABCD$ of four equal jointed rods is hanging from A, the shape being maintained by a string joining mid points of AB and BC. Prove that the ratio of tension of the string to the reaction at C is $\frac{8}{\sqrt{5}}$.

(ii) Find the conditions for a system of forces to be in Astatic equilibrium. 5+5

(b) (i) A uniform ladder of weight $2w$ inclined to the horizon at 45° rests with its upper extremity against rough vertical wall and its lower extremity on the ground. Prove that the least horizontal force which will move the lower end towards the wall is greater than $w\left[\frac{1+2\mu-\mu\mu'}{1-\mu'}\right]$, where μ and μ' are coefficient of friction of lower and upper end respectively.

(ii) Forces P, Q, R act along the lines $x = 0$, $y = 0$ and $x\cos\theta + y\sin\theta = p$. Find the magnitude of the resultant and the equation of its line of action. 5+(3+2)

Group – C

(Marks : 25)

4. Answer ***any one*** of the following :

(a) Find the loss of kinetic energy due to oblique impact of two smooth spheres of masses m_1 and m_2 moving with velocities u_1 and u_2 in same direction such that their lines of action make angle α and β with the line joining the centres of the spheres. Discuss the cases for perfectly elastic and inelastic collisions. 5+2

(b) Let a particle of mass m is moving in a straight line under an attractive force mn^2x together with a force of resistance proportional to the velocity. Discuss the cases of light damping and heavy damping. 4+3

5. Answer ***any two*** of the following :

(a) (i) Find the resultant of two S.H.M. having same period but different amplitudes along the same straight lines.

(ii) A particle is falling under the action of gravity alone. Show that the sum of the kinetic and potential energies at any point of its path is constant. 5+4

(b) (i) A heavy particle slides down a rough cycloid of which the coefficient of friction is μ. Its base is horizontal and vertex downwards. Show that if it starts from rest at the cusp and comes to rest at the vertex then $\mu^2 e^{\mu\pi} = 1$.

(ii) If a particle moves along a curve in such a way such that its tangential and normal acceleration are proportional, then find the velocity(v) in terms of ψ, where ψ is the angle the tangent to the curve makes with X-axis at time t. 6+3

(c) (i) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$.
Find the path and show that accelerations along and perpendicular to the radius vector are $\lambda^2 r - \frac{1}{r}\mu^2\theta^2$ and $\mu\theta\left(\lambda + \frac{\mu}{r}\right)$ respectively.

(ii) A particle falls to the ground from a height h. If e be the coefficient of restitution, then show that the whole distance described by the particle before it has finished rebounding is $h\frac{1+e^2}{1-e^2}$ and that the whole time taken is $\sqrt{\frac{2h}{g}}\frac{1+e}{1-e}$. (1+2+2)+2+2

(d) (i) A particle is projected under gravity in a medium whose resistance is proportional to the square of the velocity. Find the path of the particle if it is projected with velocity u at an angle 45° to the horizon.

(ii) If a particle describes the curve of equiangular spiral, find the ratio of the radial and cross-radial velocity. 6+3

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