2021

STATISTICS — HONOURS — PRACTICAL

Paper : DSE-B-1P

(Stochastic Processes and Queuing Theory)

Full Marks : 30

The figures in the margin indicate full marks.

1. The one step transition probability matrix $P = ((p_{xy}))$ of a Markov chain with state space $\{d, e, f\}$ is given below. Find (a) the 3-step transition probability matrix and (b) the steady state distribution of the chain. 2+6

$$P = \begin{pmatrix} \frac{9}{10} & \frac{1}{10} & 0 \\ 0 & \frac{9}{10} & \frac{1}{10} \\ \frac{9}{10} & 0 & \frac{1}{10} \end{pmatrix}$$

2. Keeping 4 significant digits, simulate observations from $U[0, 1]$ (as many as required), give their values, and use a suitable transformation to generate a Poisson process $(N_t)_{0 \le t \le T}$ with intensity parameter $\lambda = \frac{1}{10}$ per hour up to time $T = 75$ hours. What is the value of N_T for your process? 8+2

3. In a municipality hospital patients' arrival are considered to be Poisson with an arrival interval time of 10 mins. The doctors (examination and dispensing) time may be assumed to be exponentially distributed with an average of 6 mins.

(a) What is the chance that a new patient directly sees the doctor?

(b) For what proportion of the time the doctor is busy?

(c) What is the average number of patients in the queue and in the system?

(d) What is the average waiting time for a patient in the queue and in the system?

(e) Suppose the municipality wants to recruit another doctor, when an average waiting time of an arrival is 30 mins in the queue. Find out how large should the mean arrival rate be so as to justify a 2nd doctor. 2+2+3+3+2