2021

MATHEMATICS — HONOURS

Paper : DSE-B-1

(Boolean Algebra and Automata Theory)

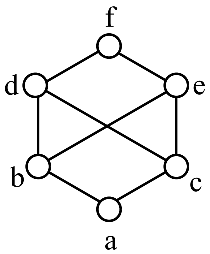
Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer ***all the*** questions : 2×10

(a) The graph given below is an example of



(i) non-lattice poset (ii) lattice

(iii) partial lattice (iv) bounded lattice.

(b) The output sequence for AND Gate with inputs $X = 111001$, $Y = 100101$, and $Z = 110011$ is

(i) 110000 (ii) 100001

(iii) 111101 (iv) None.

(c) The logic gate which gives high output for the same inputs, otherwise low output is known as

(i) NOT (ii) X-NOR

(iii) AND (iv) XOR.

(d) If $L_1 = \{a^n \mid n \geq 0\}$ and $L_2 = \{b^n \mid n \geq 0\}$, consider

(I) $L1.L2$ is a regular language.

(II) $L1.L2 = \{a^n b^n \mid n \geq 0\}$

Which one of the following is correct?

(i) Only I (ii) Only II

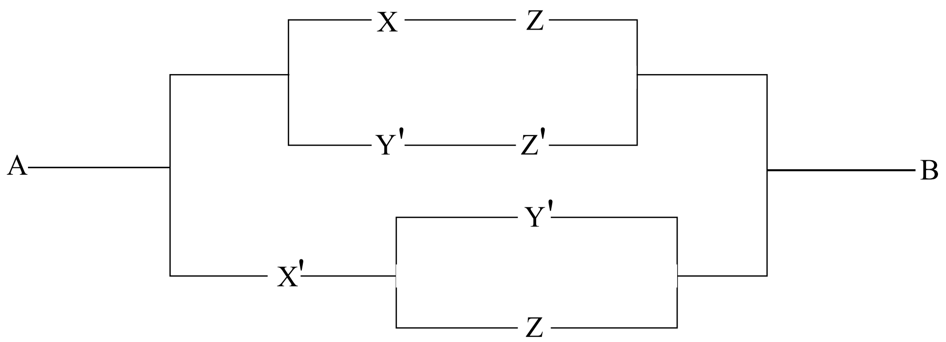
(iii) Both I and II (iv) Neither I nor II.

(e) Given Language : $L = \{ab \cup aba\}^*$ If X is the minimum number of states for a DFA and Y is the number of states to construct the NFA, $|X - Y| = ?$

(i) 2 (ii) 3

(iii) 4 (iv) 1.

(f) The Boolean function representing the following switching circuit is



(i) $(y + z)\cdot(y' + z')\cdot x' + (y'\cdot z)$ (ii) $(y + z) + (y' + z') + x'\cdot(y' + z)$

(iii) $(y\cdot z + y'\cdot z') + x'\cdot(y' + z)$ (iv) $(y\cdot z + y'\cdot z') + x'\cdot(y'\cdot z)$.

(g) Let $(L, \vee, \wedge)$ be a lattice and $S \subseteq L$. Then $(S, \vee, \wedge)$ is called a sublattice of $(L, \vee, \wedge)$

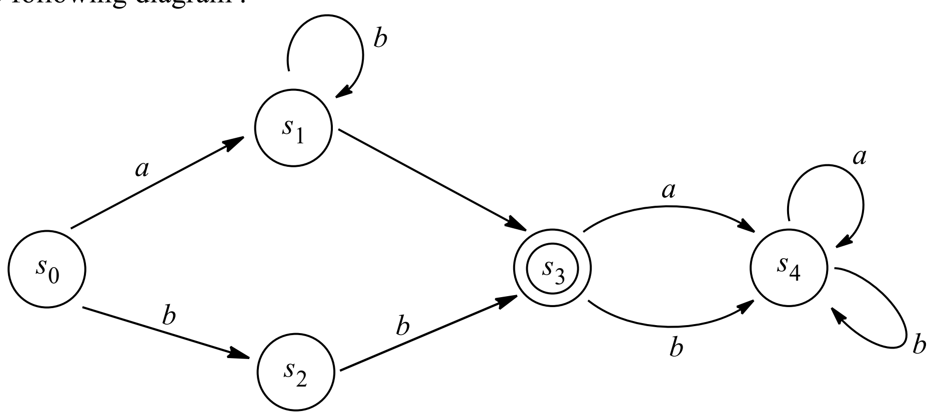
(i) if and only if S is closed under the operation $\vee$ only

(ii) if and only if S is closed under the operation $\wedge$ only

(iii) if and only if S is closed under both the operations $\vee$ and $\wedge$

(iv) if all the above statements are correct.

(h) In the following diagram :



(i) s_0 is the sink state

(ii) s_1 is the sink state

(iii) s_3 is the sink state

(iv) s_4 is the sink state.

(i) Suppose a Push Down Automaton (PDA) has the following inputs :

(I) 10101010101

(II) 101011110

Then which of the following statements is correct?

(i) Both (I) and (II) will be accepted

(ii) None of (I) and (II) will be accepted

(iii) (I) will be accepted but (II) will be not

(iv) (II) will be accepted but (I) will be not.

(j) Which is the shortest string that is not in the language represented by the regular expression $(RE) : a^*(ab)^*a^*$?

(i) a (ii) ab

(iii) ba (iv) None of these.

Unit - I

Answer ***any one*** question.

2. Prove that the direct product of any two distributive lattices is a distributive lattice. 5

3. (a) Prove that every distributive lattice is modular.

(b) Show that D_{30} is isomorphic to B_3, where D_n denotes the set of all divisors of n and B_n is the set of n tuples whose members are either 0 or 1. Can we say that D_{30} is a Boolean algebra? Justify your answer. 2+(2+1)

Unit - II

Answer ***any two*** questions.

4. A bulb in a staircases has two switches, one switch being at the ground floor and the other one at the first floor. The bulb can be turned ON and also can be turned OFF by one of the switches irrespective of the state of the other switch. Draw the switching circuits. 5

5. Simplify the following Boolean polynomials :

(i) $xy + xy' + x'y$

(ii) $xy' + x(yz)' + z$ 2+3

6. Find the minimal forms for $x_3(x_2 + x_4) + x_2x_4' + x_2'x_3'x_4$ using the Karnaugh diagrams. 5

7. Show that NOR gate is a universal gate. 5

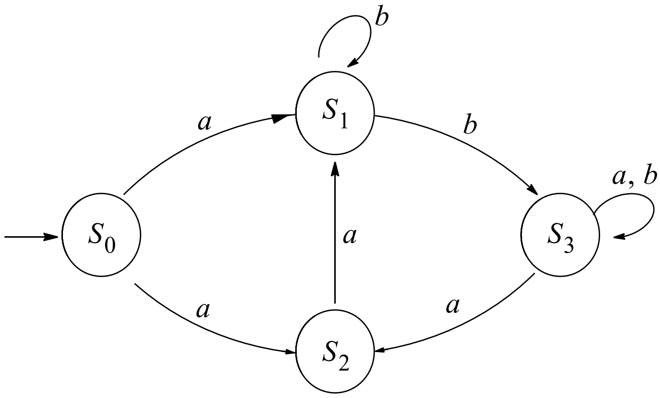
Unit - III

Answer ***any two*** questions.

8. Show that the following languages are not regular :

$L = \{a^m b^n \mid m, n > 0 \text{ and } n < m\}$. 5

9. Find a deterministic automaton which accepts the same language as the non-deterministic automaton. 5

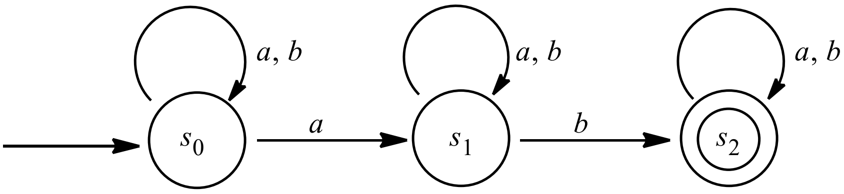


10. Find the output string corresponding to an input string *aabbb* for the FSM, where

$Q = \{0, 1\}$, $\sum = \{a, b\}$, $f : Q \times \sum \to Q$, $g : Q \times \sum \to$ O (output) and f, g are defined by : 5

	f		g	
$Q \setminus \Sigma$	a	b	a	b
q_0	q_0	q_1	0	1
q_1	q_1	q_1	1	0

11. (a) Find the language with regular expression accepted by the following automaton :



(b) State if the above automaton is a DFA or NDFA. Justify your answer.

(c) Is a regular expression unique for a language? Give example(s) in support of your answer.

1+2+2

Unit - IV

Answer ***any two*** questions.

12. Show that the following grammar is ambiguous.

$S \to aSbS \mid bSaS \mid \lambda$ 5

13. Show a derivation tree for the string aabbbb with the grammar

$$S \to AB|\lambda$$
$$A \to aB$$
$$B \to Sb$$

Give a verbal description of the language generated by this grammar. 5

14. Convert the grammar with the following production rules to Greibach Normal Form (GNF) :

$$S \to XA|BB$$
$$B \to b|SB$$
$$X \to b$$
$$A \to a$$

5

15. Construct or simulate a push down automaton (PDA) for the language $\{0^n 1^m 0^n : m,n \geq 1\}$. 5

Unit - V

Answer ***any one*** question.

16. Consider the program that simply moves the position of the machine on the tape from the beginning to the end of a string. The alphabet is $\sum = \{a, b\}$ and symbol $\Gamma = \{a, b, \#\}$, the set of states $Q = \{s_0, s_1, \#\}$ and the set of rules is given by :

(s_0, a, s_1, a, R) $(s_0, b, s_1, b, R)(s_1, a, s_1, a, R)(s_1, b, s_1, b, R)(s_1, \#, h, \#, \#)$ where the symbols carry usual meanings.

Write down the configurations of the Turing Machine (TM) from the beginning of the program to the last state. 5

17. Construct TM (Turing machine) for the language $L=\{0^n 1^n\}$, where $n >= 1$. 5

Unit - VI

Answer ***any one*** question.

18. (a) Prove that if the halting problem were decidable, then every recursively enumerable language would be recursive, consequently the halting problem is undecidable.

(b) Let $A = \{001, 0011, 11, 101\}$ and $B = \{01, 111, 111, 010\}$.

Does the pair (A, B) have a post correspondence solution? 3+2

19. Prove that there exists no algorithm for deciding whether or not

$L(G_1) \cap L(G_2) = \varphi$ for arbitrary Context–free grammars G_1 and G_2. 5