2021

PHYSICS — HONOURS

Paper : DSE-B-1(a)

(Astronomy and Astrophysics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***question nos.*** **1** and **2**, and ***any four*** questions from the rest.

1. Answer ***any five*** questions : 2×5

(a) The parallax angle for a star is 0.379″. Determine the distance modulus for it.

(b) The luminosity of the Sun is $L_\odot = 3.839 \times 10^{26}$ W. Calculate the amount of that radiant flux received at Earth in units of W m^{-2}. You can ignore the absorption by the Earth's atmosphere.

(c) Betelgeuse, a red supergiant in the constellation of Orion, has a mass about 10 $M_\odot$ and a radius about 1000 $R_\odot$. Calculate its average density given that the density of Sun is about 1410 kg m^{-3}. $M_\odot = 2\times10^{30}$ kg.

(d) The Hubble Space Telescope (HST) has a 94 inch primary mirror. According to Rayleigh criterion, what would be its resolution limit (θ) when we observe at the ultraviolet wavelength of the hydrogen Lyα line, 121.6 nm?

(e) Why does helium fusion require much higher temperatures than hydrogen fusion?

(f) Schematically plot the mass-radius graph of white dwarf stars, clearly indicating the Chandrasekhar Limit on it.

(g) Give two arguments for the existence of dark matter.

2. Answer ***any three*** questions :

(a) Suppose a star has a radius 1.67 $R_\odot$ and a luminosity 25 $L_\odot$. Use this information to calculate the energy flux at the surface of that star. Then calculate its surface temperature. Take $R_\odot = 6.9598 \times 10^8$ m and $L_\odot = 3.839 \times 10^{26}$ W. 2+3

(b) Electron scattering is the primary source of opacity in the stellar core, with the Thompson scattering cross-section being $\sigma_{th} = 6\times10^{-29}$ m^2. Assuming that the solar core consists of fully ionized hydrogen of density of 10^5 kg/m^3, calculate the distance travelled by a photon between collisions. 5

(c) Express the Saha Ionization equation in terms of partial pressure p_e of electrons. Hence show that it can be expressed in a convenient form as : $ln\,(\chi p_e) = 2.5\, lnT - (5040/T)\chi - 0.48$. Here χ is the degree of ionization of H-gas and χ = 13.6 eV. Hence calculate the degree of ionization at 10,000K temperature if p_e = 100 dyne/cm^2. 3+2

(d) Starting from the basic radiative transfer equation (neglecting the effect of scattering), show that its general solution can be written as :

$$I_\nu(\tau_\nu) = I_\nu(0)e^{-\tau_\nu} + \int_0^{\tau_\nu} e^{-(\tau_\nu - \tau'_\nu)} S_\nu(\tau'_\nu)\, d\tau'_\nu$$

where τ_ν is known as the optical depth and S_ν is the *source function* defined as the ratio of emission and absorption coefficient. From the above equation, obtain the specific intensity for optically thin ($\tau_\nu \ll 1$) and optically thick ($\tau_\nu \gg 1$) medium. You can take $I_\nu(0) = 0$. 3+2

(e) Newton believed in an infinite old and static universe filled with uniformly scattering stars. However, Newton's contemporary, Edmund Halley, disagreed to that and asked why then the night sky is dark at night. This question was later addressed by a German astronomer Heinrich Olbers, and this is known as *Olbers's paradox.* Explain this paradox. Identify the wrong assumption(s) in Newton's approach and describe a way to resolve this paradox. Also mention some of the observations supporting your answer. 5

3. Suppose a star is located at a distance of 180 pc from Earth. It has a surface temperature of 28,000 K and a radius of 5.16×10^{11} cm. You can assume that the star is consisting of a spherical blackbody. Determine the following for the star :

(a) Luminosity

(b) Absolute bolometric magnitude

(c) Distance modulus

(d) Apparent bolometric magnitude

(e) Radiant flux at the star's surface

(f) Peak wavelength, λ_{max}.

You can use some of the information about the Sun : $R_\odot = 6.9598 \times 10^{10}$ cm, $T_\odot = 5770$ K, $L_\odot = 3.839 \times 10^{26}$ W and M_{bol} of Sun = +4.75. 3+2+1+1+2+1

4. We observe HI line (rest wavelength of 21.10611 cm) at 22.58173 cm for a certain galaxy which is 269 Mpc away from us.

(a) What is the radial velocity of this galaxy in km/s? You can assume that the radial velocity is much smaller compared to the speed of light.

(b) From the information above, what is the current Hubble constant? Name the relationship that you used to figure this out.

(c) What must be the maximum age (in years) of the Universe for the Hubble constant determined above? Is this consistent with the age of the globular star clusters in the Milky Way Galaxy?

(d) Based on the data for this galaxy, what is the current mass density (ρ_0) of the Universe if $\Omega_\Lambda = 0$ and the deceleration parameter (q_0) is 0.25? You can use the formula :

$$\rho_0 = \frac{3q_0 H_0^2}{4\pi G}$$

where G = 6.673×10^{-11} Nm^2 kg^{-2}.

(e) From the above data, what is the matter density parameter equal to? Is such a Universe closed, flat or open? Give explanation for your answer. 2+2+2+2+2

5. (a) From the stellar structure equation related to hydrostatic equilibrium show that the pressure profile of a model star of radius R and constant density ρ is given by $P(r) = \frac{2}{3}\pi G \rho_2 \left(R^2 - r^2\right)$.

(b) Using the scaling relations obtained from the equations of stellar structure, show that for the low-mass stars $L \sim M^5$, where opacity $\kappa \sim \rho T^{-3.5}$. Symbols have their usual meaning.

(c) Assume that the Earth's atmosphere contains only nitrogen. What should the mass of the atmosphere be for it so as to be unstable against Jeans collapse at standard temperature and pressure, i.e. T = 300 K and P = 10^6 dyn-cm^{-2}? Express it in terms of earth-mass. 3+4+3

6. (a) Consider a rotating neutron star with a mass $M = 2M_\odot$, radius R = 15 km, period P = 0.1 s, and a rate of change of the period $\dot{P} = 10^{-6}$ s yr^{-1}. Find the rate at which its rotational kinetic energy is decreasing, and the approximate lifetime of the pulsar if it loses energy at this rate.

(b) Suppose the total energy associated with a supernova explosion is due to the self-gravitational collapse of a 1 $M_\odot$ star to a radius of 10 km. If 0.01% of this energy is emitted as light over a period of one month, what is the luminosity of the supernova in the units of $L_\odot$? 5+5

7. Analytic functions can be derived for the pressure and density structure in the interior of Jupiter if an approximate relationship between pressure and density is assumed. A reasonable choice for a composition of pure molecular hydrogen is

$$P(r) = K\rho^2(r)$$

where K is a constant. This type of analytic model is known as a *polytrope.*

(a) By substituting the expression for the pressure into the hydrostatic equilibrium equation and differentiating, show that a second-order differential equation for the density can be obtained as

$$\frac{d^2\rho}{dr^2} + \frac{2}{r}\frac{d\rho}{dr} + \left(\frac{2\pi G}{K}\right)\rho = 0$$

(b) Show that the above equation is satisfied by $\rho(r) = \rho_c \left(\frac{\sin kr}{kr}\right)$, where ρ_c is the density at the centre of the planet and $k = \left(\frac{2\pi G}{K}\right)^{1/2}$.

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(c) Taking the average radius of Jupiter to be $R_J = 6.99 \times 10^7$ m and assuming that the density goes to zero at the surface (i.e. $kR_J = \pi$), determine the values of k and K. 4+4+2

8. (a) Imagine a universe filled with a pressureless "dust" of uniform density, $\rho(t)$. Using the Newtonian cosmology, show that the equation describing the time behaviour of the scale factor $R(t)$ will look like

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right] R^2 = -kc^2$$

where k is the constant curvature parameter.

(b) For such one-component universe show that the density parameter,

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} = 1 + \frac{kc^2}{(dR/dt)^2}\,.$$

where ρ_c is the critical density. What does this have to say about the nature of the early universe?

(c) Show that $dR/dt \to \infty$ as $t \to 0$. What does this say about the difference between a closed, a flat, and an open universe at very early times? 4+4+2

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