M. Sc. (Physics) 4th Semester Examination 2021
PHY 521 (Advanced Condensed Matter I

Full Marks: 50 Time: 2.5 hrs

(2 hours for answering and 30 minutes for downloading, scanning, and mailing back)

Answer any **five** questions

Instructions:

(a) Write your *Examination Roll Number* and *Registration Number* (from an earlier admit card) at the top of your answer script.

(b) Do not write your name or class roll number anywhere.

(c) Write page number on top of each page.

(d) Scan the complete answer script into *a single pdf file* and mail it to the e-mail from where you got this question paper.

(e) The answer script file for the paper PHYAAA (where AAA is the paper code like 521, 522, and so on) must be named as instructed: Note that your Examination Roll Number is of the form ZZZ/PHY/XXXXXX, where ZZZ is the college identifier (like C91, 031, etc.), and XXXXXX is a 6-digit number like 191099.

— For CU students, the filename for the paper PHYAAA must be CUXXXXXXPHYAAA.pdf. For example, the script of PHY521 coming from C91/PHS/191099 must be named CU191099PHY521.pdf.

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1. (a) Using Kubo-Greenwood formalism, show that the localized states of an amorphous material do not carry any current at absolute zero temperature. What happens at finite non-zero temperature?

(b) Explain briefly Mott's minimum metallic conductivity. Does scaling theory of localization support this in any arbitrary spatial dimension?

(c) Consider a 1D system which is characterized by the scaling function of conductance $\beta(g)$. Now, we stack 20 of these 1D systems next to each other and weakly couple them. According to scaling theory of localization, what will be the new scaling function? Justify your answer.

Assume that in the Ohmic region, the general form of the β function is

$$\beta = d - 2 - \frac{A_d}{g}$$

By integrating the above equation in the interval $[L_0, L]$, obtain the explicit length dependence of the conductivity for $d = 1$.

(3+1)+(2+1)+(2+1)

2. (a) The energy band in a linear chain with interatomic distance a is given by $E(k) = E_0 - t\cos ka$, (E_0 and t are constants). When the width of this band is increased by 20%, what is the percentage change of the effective mass of the electron at the bottom of the band?

(b) Stating clearly the assumptions, obtain an expression of Mott variable range hopping (VRH) conductivity for 2d amorphous system as a function of temperature.

(c) The resistivity of a metal in the presence of magnetic impurities, as per Kondo's calculation, is given by

$$R(T) = R_0\left[1 + 2J\rho \log \left|\frac{k_B T}{D - \epsilon_F}\right|\right]$$

where R_0 is the resistivity in absence of an impurity, J is the strength of the interaction between conduction electrons and impurity, D is the range of the electron energy, and ρ, a constant density of states. Find an estimate of the Kondo temperature T_k in terms of these parameters.

(d) Explain how "screening" occurs if an external negative charge is inserted in the uniform electron gas in the presence of a positive ion background.

2+3+3+2

3. (a) Is magnetic dipolar interaction responsible for the exchange interaction in Heisenberg Spin Hamiltonian? Explain.

(b) With the help of Holstein-Primakoff transformation, derive the magnon dispersion relation $E(ka)$ of a ferromagnetic linear chain of lattice constant a. Sketch the dispersion relation as a function of dimensionless varable ka.

(c) Three quantum $S = 1$ atoms are located at the vertices of an equilateral traingle and is given by the Hamiltonian $H = -2J\left(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1\right)$. Find the energy eigenvalues of the system.

(d) Some organic molecules have a triplet ($S = 1$) excited state at an energy $k_B\Delta$ above a singlet ($S = 0$) ground state. Draw the energy levels of the system in presence of a magnetic field B. Hence. explain why the system does not show spontaneous magnetization at a given temperature T.

2+(3+1)+2+2

4. (a) Distinguish between the energy gap in superconductor and that in semiconductor. Below is a list of critical temperature T_c (measured in K) and the energy gap $\Delta(0)$ (measured in 10^{-4} eV) at zero temperature. Verify whether they satisfy BCS theory or not.

System	Sn	Al	In	Tl
$\Delta(0)$	5.614	1.753	5.278	3.71
T_c	3.72	1.196	3.40	2.39

(b) In the mean field level (HF theory), the following matrix elements in connecton with the BCS theory of superconductivity can be simplified as

$$\left\langle \Psi_{BCS} \right| \sum_{kk'} \; V_{kk'} b_k^\dagger b_{k'} \left| \Psi_{BCS} \right\rangle \approx \sum_{kk'} \; V_{kk'} \left\langle \Psi_{BCS} | b_k^\dagger | \Psi_{BCS} \right\rangle \langle \Psi_{BCS} | b_{k'} | \Psi_{BCS} \rangle$$

Evaluate the matrix elements and physically interpret the result.

(c) Which particular continuous symmetry is spontaneously broken in superconductivity? What is its consequence?

(2+2)+(3+1)+(1+1)

5. (a) Ψ_A and Ψ_B are two determinantal states, describing a system of three fermions.

$$\Psi_A = \frac{1}{\sqrt{3!}} \det\{\psi_1 \psi_2 \psi_3\}$$

$$\Psi_B = \frac{1}{\sqrt{3!}} \det\{\psi_1 \psi_2 \psi_4\}$$

Here ψ_1, ψ_2, ψ_3, ψ_4 are single-particle wave functions.

Given that $G_1 = \sum_i h_i$ and $G_2 = \frac{1}{2} \sum_{i,j;i \neq j} u_{i,j}$ are one-body and two-body operators respectively, evaluate $\langle \Psi_A | G_1 | \Psi_B \rangle$ and $\langle \Psi_A | G_2 | \Psi_B \rangle$.

(b) Discuss why Hartree-Fock method provides an upper bound to the actual ground-state energy of a many-electron system.

(4+4)+2

6. (a) Consider a many-body fermionic state $|1100110 \cdots\rangle$. Express this state in terms of excitations (i) about the vacuum state $|0000000 \cdots\rangle$; and (ii) about the "Filled Fermi sea" $|1111000 \cdots\rangle$.

(b) For a many-fermion system, the density operator is given by

$$\rho(\mathbf{r}) = \Psi^\dagger(\mathbf{r})\Psi(\mathbf{r})$$

Where $\Psi(\mathbf{r})$ is the fermionic field. Expanding $\Psi(\mathbf{r})$ in the plane wave basis, compute the Fourier transform of $\rho(\mathbf{r})$.

(c) An "extended" version of the Hubbard Hamiltonian is given by :

$$H = -t \sum_{<jl>,\sigma} c^\dagger_{j\sigma} c_{l\sigma} + U \sum_j n_{j\uparrow} n_{j\downarrow} + V \sum_{<jl>} n_{j\uparrow} n_{l\downarrow}$$

Here, in addition to the usual on-site interaction U, there is an interaction V between the up-spin and down-spin electrons belonging to nearest neighbour sites.

(i) Write down the possible states for such a 2-site system comprising of one up-spin electron and one down-spin electron.

(ii) If the hopping t is set to 0, construct the Hamiltonian in matrix notation and find the corresponding energy eigenvalues.

2+3+(2+3)

7. (a) Consider a cylinder of radius R and height Z, containing superfluid Helium. It is rotating with an angular velocity Ω. Vortex lines are formed inside the superfluid.

(i) Show that the total number of vortices increases with an increasing angular velocity Ω.

(ii) For a very large angular velocity, the vortices start to overlap. Estimate the value of this critical Ω_c in terms of the vortex core radius a_0. Argue whether the system would remain a superfluid if $\Omega > \Omega_c$.

(b) Draw the occupation probability $P(p)$ vs. momentum p curve for superfluid ^{4}He, both for $T < T_c$ and $T > T_c$ (T_c is the critical temperature). How is it different from the momentum distribution of a non-interacting Bose gas?

(3+3)+(2+2)