2021

STATISTICS — HONOURS

Third Paper

(Group - A)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Section - I

Answer ***any two*** from ***question nos.*** **1-4** and ***any one*** from ***question nos.*** **5** and **6**.

1. Establish the relationship between E and Δ.

Show that $\Delta^n U_{x-n} = U_x - \binom{n}{1} U_{x-1} + \binom{n}{2} U_{x-2} - \binom{n}{3} U_{x-3} + \ldots + (-1)^n U_{x-n}$. 2+3

2. Stating clearly all the assumptions, derive trapezoidal's rule for numerical integration. 5

3. Derive the form of the error term in Newton's forward formula. 5

4. Find the extrema of the function $f(x, y) = 2y + x$, subject to the constraint $0 = g(x, y) = y^2 + xy - 1$. 5

5. Describe Newton-Raphson method for solving single unknown. Discuss about its geometric significance and convergence. 8+4+3

6. (a) What do you mean by transformation of variables? Discuss, with examples, the role of Jacobian in this context.

(b) Discuss the Lagrange multiplier method. For a rectangle, whose perimeter is 20 m, use this to find the dimensions that will maximize the area. (3+5)+(3+4)

Section - II

Answer ***any two*** from ***question nos.*** **7-10** and ***any one*** from ***question nos.*** **11** and **12**.

7. If $P(t)$ is the probability generating function of a non-negative integer valued random variable X, with $q_j = P[X > j]$, then show that the generating function of q_j is given by,

$$Q(t) = \frac{1 - P(t)}{1 - t}, \ |t| < 1$$ 5

8. What do you mean by loss of memory property of a statistical distribution? Name two such distributions. 5

9. Discuss Thurstone scaling procedure. 5

10. Suppose a $N(\mu, \sigma^2)$ distribution is left truncated at $x = A$. Find the mean of this distribution. 5

11. (a) Show that the hyper geometric distribution originates as a result of random sampling without replacement from a finite population.

(b) Show that the cumulative probabilities of a binomial distribution can be expressed as incomplete Beta functions. 7+8

12. (a) 'If X and Y are two random variables with correlation coefficient ρ, such that (X, Y) follows bivariate normal distribution, then X and Y are independent iff $\rho = 0$'— prove or disprove.

(b) Let X be a continuous random variable symmetrically distributed about 'a'. Let Y be another random variable, such that,

$$Y = \begin{cases} 1, & \text{if } x \leq a \\ 0, & \text{if } x > a \end{cases}$$

Let $Z = |X - a|$, show that Y and Z are independently distributed.

(c) For a normally distributed random variable, show that all odd order central moments are zero. 4+7+4