

2022

MATHEMATICS — HONOURS

Paper : SEC-B-2

(Scientific Computing with SageMath/R)

Full Marks : 80

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.***(Answer either Scientific Computing with SageMath or Scientific Computing with R)****[Scientific Computing with SageMath]***Notations and symbols have their usual meaning.**Throughout the question paper sage code means code in SageMath.*

1. Choose the correct alternative (Give justification whenever it is applicable). 2×10
- (a) The output of $\text{sqrt}(144)$ and $\text{sqrt}(8)$ are
- | | |
|-----------------------------------|--|
| (i) 12 and $\text{sqrt}(8)$ | (ii) $4 * \text{sqrt}(9)$ and $\text{sqrt}(8)$ |
| (iii) 12 and $2 * \text{sqrt}(2)$ | (iv) none of these. |
- (b) What will be the output of the following sage code?
- $$\text{log}(\text{sqrt}(100 \wedge 3), 10)$$
- | | |
|--|------------------------------------|
| (i) $\text{log}\left((100)^{3/2}, 10\right)$ | (ii) $\text{log}(10 \wedge 3, 10)$ |
| (iii) 3 | (iv) none of these. |
- (c) The output of $19/2 - 19//2 + 19\%2$ in sage is
- | | |
|-------------|-----------|
| (i) 1.5 | (ii) 1 |
| (iii) $3/2$ | (iv) 9.5. |
- (d) What happens when `'2' == 2` is executed?
- | | |
|------------------------|------------------------|
| (i) False | (ii) ValueError occurs |
| (iii) TypeError occurs | (iv) True. |

Please Turn Over

(e) What will be the output of the following sage code?

```
for i in range(2.0) :
```

```
    print(i)
```

(i) 0.0 1.0

(ii) 0.1

(iii) An error will occur

(iv) none of these.

(f) Which of the following is the correct code for computing all divisors of 24?

(i) divisors(24)

(ii) divisor(24)

(iii) *divisor_all*(24)

(iv) divisors(24, all).

(g) What will be the output of the following sage code?

```
f(x) = 3 * x + 5
```

```
g(x) = x ^ 3 + 1
```

```
h(x) = f(g(x))
```

```
print h(x)
```

```
print h(1)
```

(i) $3 * x^3 + 8$

(ii) $3 * x^3 + 5$

11

32

(iii) $(3 * x + 5)^3 + 1$

(iv) none of these.

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(h) To evaluate $\sqrt{-4}$, which one of the following code is correct?

(i) `sqrt(-4)`

(ii) `sqrt(-4, all=true)`

(iii) `sqrt(-4, complex=true)`

(iv) `sqrt(-4).n(all)`.

(i) Which one of the following sage commands is not correct to get the derivative of $\sin(x^2)$?

(i) `diff(sin(x^2), x)`

(ii) `g(x) = sin(x^2)`

```
diff(g(x), x)
```

(iii) `g(x) = sin(x^2)`

(iv) `g(x) = sin(x^2)`

```
g.derivative(2)
```

```
print("g'(x)=", diff(g(x), x))
```

(j) Which one of the following sage codes is correct?

(i) a=5

(ii) a=5

(iii) a=5

(iv) a=5

b=7

b=7

b=7

b=7

```
if(a>b) :
```

```
if(a>b)
```

```
if(a>b) :
```

```
if(a>b) :
```

```
    print(a)
```

```
    print(a)
```

```
    print(a)
```

```
    print a
```

```
else :
```

```
else
```

```
else :
```

```
else :
```

```
    print(b)
```

```
    print(b)
```

```
    print(b)
```

```
    print b
```

Section - IAnswer **any one** question.

2. (a) Give an example to show sage can be used as a calculator.
 (b) What will be the output of the following sage code? State the reason :

```
r = 30
area = π*r^2
print(area)
```

- (c) Write the sage commands to calculate the expression

$$500 \frac{[1 - (1 + 0.05)^{-30}]}{0.05}$$

- (d) Write built-in sage commands to find the common logarithm of $\sqrt{100^3}$. Justify that sage will give the exact answer. 2+2+2+(2+2)
3. (a) Write sage commands to evaluate the following expressions :

Value of $\sqrt[20]{e} + \sqrt[2]{20}$ in 20 decimal places, the absolute numerical value of $\frac{\pi}{\frac{22}{7}} - 1$ in 50 decimal

places, the value of $\sqrt{3 + \sqrt{3 + \sqrt{3 + \frac{1}{3}}}}$ in 10 digits.

- (b) Give the output of the following sage commands.

$27 \wedge (1/3)$, $64 \wedge (1/4)$, $(2 \wedge 10) \wedge (1/3)$, $8 \wedge (1/3) + 16 \wedge (1/4)$.n() (2+2+2)+(1+1+1+1)

Section - IIAnswer **any one** question.

4. (a) How can you plot functions with their asymptotes in sage? Explain for the function $f(x) = \tan(x)$ for $-\Pi < x < \Pi$.
 (b) Plot the graph of $f_n(x) = \sin^n(x)$ in sage in the interval $[0, \pi]$ for $n = 1, 2, \dots, 4$. Use different colors and different plot styles for different graphs. 5+5

5. (a) Write sage code to plot a 5-petaled rosette,
 $r(\theta) = 2 \cos 5\theta, 0 < \theta < 2\pi$

In the above plotting which one is the input and which one is the output?

- (b) Correct the following sage code :

```
plot(sin(t), -10, 10)
```

- (c) How can you plot piecewise functions in sage? Give examples. (2+2)+2+4

Please Turn Over

Section - III

Answer *any one* question.

6. (a) Write sage code to find the first two derivatives of $f(t) = \ln(1+t^2)$ and plot them along with the graph $f(t)$. Use different colors for different graphs.

(b) Differentiate $f(x) = \frac{1}{x^2}$ with sage using the definition of derivative in $(0, \infty)$ 5+5

7. (a) Write sage code to evaluate $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx$.

(b) Write sage code to evaluate $\int_0^1 \frac{\sqrt{x + \sqrt{x^2 + 1}}}{(x+1)} dx$.

(c) Write sage code to integrate the function $\sin(x^2 + y^2)$ with respect to x and print the result.

3+3+4

Section - IV

Answer *any one* question.

8. (a) Write a program in sage to find the area of a circle whose perimeter is P in 30 decimal places.
 (b) Write a program in sage without using inbuilt function to print first 20 non-fibonacci positive integers. 4+6

9. (a) Explain **for loop** construction giving an appropriate example in SageMath.
 (b) Write a sage function to check whether a number taken from the user is prime or not.
 (c) Find the syntax error in the following code written in sage and correct it.

```
x = 1
while x ≤ 10
print x * 5
```

2+5+3

Section - V

Answer *any one* question.

10. Give sage codes to do the following :

- (a) Define a matrix A whose rows are (11, 30, 40, 70), (30, 40, 70, 90), (30, 70, 0, 101), (10, 30, -40, 80).
 (b) Display the matrix and find the row sums and column sums.
 (c) Compute the determinant of the matrix.
 (d) Find $B = AA^t$ and $C = A - A^t$.
 (e) Test to show that the matrix B is symmetric and C is skew-symmetric.

2+(1+1+1)+1+(1+1)+(1+1)

11. (a) Write sage code to find the solution of the differential equation $\frac{dy}{dx} = 3x^2$. Write also the output.

Write the sage code to plot the solution curves corresponding to three suitable choices of the arbitrary constants.

(b) Write sage code to solve the following initial value homogeneous (IVP) :

$$4 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + 17; y = 0, y(0) = -1, y'(0) = 2$$

Also plot the solution curve.

(2+1+3)+4

Section - VI

Answer *any one* question.12. (a) Write a sage program to yield a numerical approximation of u_n for $n = 20$, where $u_0 = 1$,

$$u_{n+1} = \frac{1}{1+u_n^2}, n \in \mathbb{N}$$

(b) Using in-built function in sage find the mode of the following numbers :

1, 1, 2, 3, 1, 3, 3, 5. Also find how many times the mode has occurred.

(c) Find output of the following two sage codes :

```
(i) sage : s = 0
sage : for k in [1..3]
sage :     s = s+k
sage :     s = 2*s
sage :     s
```

Please Turn Over

(ii) sage : $s = 0$
 sage : for k in [1..3]
 sage : $s = s+k$
 sage : $s = 2*s$
 sage : s

4+4+(1+1)

13. (a) Using sage code, create an adding machine that will keep on asking the user for numbers, add them together and show the total after each step. Terminate when user enters a zero.

(b) Write sage code to solve the following polynomial equation :

$$3^2(x^2 + x + 1) - x^2 - 3x + 2 = 0$$

(c) Write sage code to compute the number of zeros and the number of ones present in $\log(2022!)$ (where 2022! means factorial of 2022). 4+3+3

[Scientific Computing with R]

1. Answer **all** questions with proper explanation / justification (**one** mark for correct answer and **one** mark for justification whenever applicable) : 2×10

(a) What should we type to get the value of $\sin(45^\circ)$ in R?

(i) $\sin 45^\circ$

(ii) $N\left(\sin\left(\frac{\pi}{4}\right)\right)$

(iii) $\sin\left(\frac{\pi}{4}\right)$

(iv) none of these.

(b) What will be the output of the following R code?

Options (digits = 4) 10/3

(i) 3.33

(ii) 3.333

(iii) 3.3333

(iv) none of these.

(c) What is the output of the code given below?

A = 10

B = 20

print (A, B)

(i) 10

(ii) 20

(iii) 10 20

(iv) Error.

(d) What is the meaning of "<-"?

(i) Functions

(ii) Loops

(iii) Addition

(iv) Assignment.

(e) Which of the following code generate a sequence of integers from 1 to 10?

(i) `> for (i in 1 : 9){
+ print (i)
+}
[1]`

(ii) `> for (i in 0:9){
+ print(i)
+}
[1]`

(iii) `for (i in 1 : 10){
print(i)
}`

(iv) `for (i in 2 : 50){
print(i)
}`

(f) What is the output of `trunc(- 9.8)` in R?

(i) - 10

(ii) - 9

(iii) 10

(iv) 9

(g) Which of the following is the correct outcome of the following?

`x = 3`

`while (x<15) {x=5*x-10; if (x==20) break; print (5*x);}`

(i) `[1] 25
[1] 75`

(ii) `[1] 5
[1] 25
[1] 50
[1] 75`

(iii) `[1] 15
[1] 30
[1] 45
[1] 60
[1] 75`

(iv) none of these.

(h) Which of the following is the correct output of `Y[X[3]]` and `X[Y[7]]` respectively of the following commands :

`Y = C (42, 53, 33, 45, 61, 27, 30)`

`X = C (3, 1, 2, 4, 7, 9, 8)`

(i) 33 and 8

(ii) 53 and NA

(iii) 2 and 30

(iv) none of these.

(i) What will be the outcome of the following command?

`C(1/243, 1/81, 1/64) ^ C(- 1/5, - 1/4, - 1/2)`

(i) 3 3 8

(ii) 1/243 1/81 1/64

(iii) NaN NaN NaN

(iv) Error.

(j) What should we type to find the value of $\int_0^1 (x^3 + 2x) dx$ in R?

- | | |
|---|---|
| (i) f<- function (x) x^3+2*x
integrate (f, 0, 1) | (ii) f<-function (x^3+2*x)
integrate (f, 0 to 1) |
| (iii) f<- x^3+2*x
integrate (f, 0, 1) | (iv) f<- x^3+2*x
integrate (f, 0, 1) |

Section - I

Answer *any one* question.

2. Find the outputs of the following R commands :

- (a) a = 7; b = 6; a = a^3-b^2; b = (a+3); b
 (b) a = 1 : 5; a = a/2; a
 (c) a = 20; a = (a/2)^3; a = (a)^(1/3); a
 (d) A = matrix (c(2, 4, 6, 7), nrow = 2);
 B = matrix (c(3, 5, 7, 8), nrow = 2);
 X = A+B; X

2+2+2+4

3. Find the outputs of the following R commands :

- (a) x = 5:3; x = x^2+1; x
 (b) x = 7:3; x = x^2+2; x = length(x); x
 (c) A = matrix (C(1, 2, 3, 4), nrow = 2);
 B = matrix (C(4, 3, 2, 1), nrow = 2)
 X = A%%B; X.

3+3+4

Section - II

Answer *any one* question.

4. (a) Write a R code to plot graph of the function $f(x) = e^{x^2} + \log_e x - \tan^{-1} x$ in the range (3, 5) with blue color and with proper x-label and y-label.
 (b) Let, runs of a batsman in last 5 innings are : 51, 18, 99, 76 and 05. Write a R code to compare his runs per innings using a pie-chart, bar diagram and scatter plot, in a single figure. 5+5

5. Write R codes for plotting

(a) the graph of polar curve $r = 1 + \cos\theta$, $0 \leq \theta \leq \pi$.

(b) the following functions in one plot :

$$f(x) = \begin{cases} 1+x, & -1 \leq x \leq 0 \\ 1+x^2, & 0 < x < 1 \end{cases}$$

5+5

Section - III

Answer *any one* question.

6. Write R code to do the following for the function $f(x) = (x-1)^5$ in $[-2, 2]$:

(a) find $f''(x)$

(b) find $f'''(x)$

(c) plot $f''(x)$ and $f'''(x)$ in $(-2, 2)$ with red and green colors respectively in one plot. 3+3+(2+2)

7. (a) Write a R program to tabulate values of the function $f(x) = x^5 - 9x + 3$ along with its derivative in the interval $[-5, 5]$ with the increment value 0.2.

(b) (i) $\int_0^3 (5x^3 + 3x) dx$

(ii) $\int_0^{\infty} \frac{3}{\sqrt{\pi}} e^{-x^2} dx$

(iii) $\int_0^1 \frac{\sin x}{x} dx$

5+(1+2+2)

Section - IV

Answer *any one* question.

8. (a) Without using inbuilt functions, write R code to find the geometric mean and mode of the data set C (2, 1, 3, 2, 5, 3, 2, 4, 2, 1, 6).

(b) Without using inbuilt functions write a program in R to determine the total number of primes less than x , print the list of such primes and get an output for $x = 79$. (3+2)+(3+1+1)

9. (a) Write a R program to find sum of $1^3 + 2^3 + \dots + 100^3$.

(b) Write a R program to find the factors of 100 and find the sum of the factors.

5+5

Please Turn Over

Section - VAnswer *any one* question.

10. Given the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, write R program to

- (a) check if A is orthogonal.
 (b) check if inverse of A exist, and if so, check if $A = A^{-1}$.
 (c) select even elements of A and change them to 0. 3+4+3
11. (a) Define a matrix in R whose rows are (1, 7, 4, 3), (3, 5, 8, 9), (3, 7, 1, 1), (1, 2, 1, 2).
 Give R code to find the row reduced echelon form of the matrix.
 (b) Solve the following system of equations by a R program :

$$\begin{aligned} 2x + y + z &= 7 \\ x + y - 8z &= -5 \\ 3x - 4y + 5z &= 14 \end{aligned}$$

5+5

Section - VIAnswer *any one* question.

12. (a) Write R code to solve the following differential equation with the boundary conditions :

$$x_0 = 0, y(x_0) = 1, x_1 = \frac{\pi}{2}, y(x_1) = 4$$

$$\frac{d^2y}{dx^2} + y = 0$$

- (b) (i) Write a R code to solve the differential equation $\frac{dy}{dx} = \sin x$.
 (ii) Write a program in R to find the greatest of three given integers a, b, c . 5+(2+3)
13. (a) Write a R program to find roots of a quadratic equation $ax^2 - bx - c = 0$.
 (b) Write a R program to find area of a square inscribed in a circle of radius r . 5+5
-