2022

PHYSICS

Paper : PHY 521

(Advanced Condensed Matter Physics-I)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

1. (a) Explain why the free electron gas model of the valence electrons in a metal is not suitable to explain cohesive energy.

(b) Consider a system of N interacting electrons. Write down the expressions for (i) E_o^{HF} (the total ground state energy of the system under Hartree-Fock approximation), and (ii) ϵ_i (the single-particle energies) in terms of the single-particle wave-functions ψ_i. Discuss why E_o^{HF} is not equal to the sum of ϵ_i.

(c) In the system described above, an electron is now removed from the occupied spin-orbital ψ_m and is transferred to the previously unoccupied spin-orbital ψ_n. All the other occupied orbitals remain unchanged. Calculate the corresponding transition energy.

2+(2+2)+4

2. (a) In a particular representation, let the operator a be expressed as $\frac{1}{2}(\sigma_1 - i\sigma_2)$, and $a^\dagger$ as $\frac{1}{2}(\sigma_1 + i\sigma_2)$. Here σ_1 and σ_2 are the Pauli spin matrices, satisfying the anti-commutation relation $\{\sigma_j, \sigma_k\} = 2\delta_{jk}I$. Prove that a and $a^\dagger$ satisfy fermionic anti-commutation rules.

(b) The Hartree-Fock ground state energy for a system of N electrons is given by

$$E_0 = N\Big(\frac{3}{5}\frac{\hbar^2 k_F^2}{2m} - \frac{3}{4}\frac{e^2 k_F}{\pi}\Big)$$

Show that at a sufficiently low density, a spin-polarized ferromagnetic state of electrons becomes more stable than its usual unpolarized counterpart. You can use the following : Bohr radius $a_B = \hbar^2/me^2$, $k_F a_B = 1.92/r_s$, where r_s is a dimensionless parameter such that $r_s a_B$ is the radius of the sphere containing 1 electron in average.

5+5

3. (a) Assume that the superfluid wavefunction $\Psi(r) = \sqrt{n(r)}e^{i\theta(r)}$. Find the current density of the flow and the superfluid velocity.

(b) A container of superfluid Helium is maintained at temperature T_1 and pressure P_1, while another such container is maintained at temperature T_2 and pressure P_2. Next, these two volumes are joined by a thin capillary. Explain whether the temperatures of the containers would come to an eqilibrium or not.

(c) A non-interacting bose gas follows the dispersion relation $\epsilon(p) = p^2/2m$, while, for a weakly interacting Bose gas it is $\epsilon(p) \approx p^2/2m + gn$. Here n is the particle density, and g is the interaction strength. Using Landau's criterion for superfluidity, find out which among these two systems can support a superflow.

3+3+4

4. (a) What are the possible states of a single-site Hubbard model? Construct the partition function and find out the average occupancy $\langle n \rangle$ for this system. Argue why there would be a jump in the chemical potential when the filling crosses $\langle n \rangle = 1$.

(b) Consider the Landau free energy for the superconductor-metal phase transition as

$$F_S = F_N(0) - AT^{\gamma} + \alpha|\Psi|^2 + \beta|\Psi|^4$$

Here, A and β are constants and α is a function of temperature.
What value of γ is allowed for superconductor-metal phase transition? What signs of α and β do you prescribe to study the above phase transition? Hence, discuss the relevant symmetry being broken spontaneously in such a phase transition with the help of a diagram.

(1+3+1)+(1+1+3)

5. (a) Briefly explain the origin of exchange interaction in a magnetic system.

(b) Justify Holstein-Primakoff transformation used in spin wave theory. Show that the usual commutation relation between spin raising and lowering operator is obeyed.

(c) Consider the following expression of magnetization $M(T)$ of ferromagnetic spin waves having dispersion $\omega(q)$ at any finite temperature in an arbitrary dimension d as

$$M(T) = M(0) - \frac{V}{(2\pi)^d} \int \frac{d^d q}{e^{\beta\omega(q)} - 1}$$

Show that for $d \leq 2$, magnetization is destroyed at any non-zero finite temperature. However, for $d = 3$, show that $M(T) = M(0)\left[1 - \left(\frac{T}{T_0}\right)^{3/2}\right]$, where T_0 is a temperature scale.

3+(2+2)+3

6. (a) Reduce the BCS Hamiltonian to Anderson's Pseudo-spin form. Why is it called Pseudo-spin method?

(b) The energy difference (W) between the superconducting and normal state in BCS theory is given by

$$W = \sum_k |\xi_k| - \sum_k \frac{\xi_k^2}{E_k} - \frac{\Delta^2}{2} \sum_k \frac{1}{E_k}, \quad \xi_k = \epsilon_k - \mu, \quad E_k = \sqrt{\xi_k^2 + \Delta^2}$$

(where the symbols have their usual meanings.)
Show that in the continuum limit with $|\xi_k| \leq \hbar\omega_D$ under weak coupling approximation, W reduces to $-\frac{1}{2}N(E_F)\Delta^2$, where $N(E_F)$ is the DOS at the Fermi energy and 2Δ is the energy gap.

(c) Obtain an expression of the coherence length of the superconductor from Uncertainty principle and hence, estimate its value for a superconductor having Fermi velocity $v_F = 2.6 \times 10^6\ ms^{-1}$ and transition temperature $T_C = 4.2\ K$.

(3+2)+3+2

7. (a) Distinguish between localized states and delocalized states. Using scaling theory of localization, show that there is no minimum metallic conductivity in $d = 3$.

(b) Consider the Hamiltonian

$$H = -t\sum_i(c^\dagger_{i+2}c_i + h.c) + V\sum_i\ c^\dagger_i c_i$$

defined on a one dimensional lattice (of lattice constant a). Here t and V are constants. Obtain an expression of the density of states $N(E)$ as a function of energy E and sketch it. Find the value of the effective mass of the electron at the bottom of the band following the above Hamiltonian.

(2+3)+(3+1+1)
