2021

STATISTICS — HONOURS

Third Paper

(Group - B)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Section - I

Answer ***any two*** from ***question nos.*** **1-4** and ***one*** from ***question nos.*** **5** and **6**.

1. Let $U_{(1)}, U_{(2)}, ..., U_{(n)}$ be the order statistics of a sample of size n from $U(0, 1)$. Show that

$$E\left(U_{(r)}\right)=\frac{r}{n+1} \text{ and } Var\left(U_{(r)}\right)=\frac{r(n-r+1)}{(n+1)^2(n+2)}.$$ 5

2. Show that sample mean and sample variance are independently distributed when iid sample units are drawn from $N(\mu, \sigma^2)$. 5

3. If $X, X_2, ..., X_n$ be iid random observations from $N(0,1)$ and $S_n = \sum_{i=1}^{n} X_i^2$, then show $S_n \sim \chi_n^2$. 5

4. Define t distribution with n degrees of freedom. Write down its properties. 5

5. (a) X and Y be two independent normal variables with zero means. Show that $U = \frac{XY}{\sqrt{X^2+Y^2}}$ is normal.

If $var(X) = var(Y)$, show that $V = \frac{X^2-Y^2}{\sqrt{X^2+Y^2}}$ is also normal. Moreover, show that U and V are independent.

(b) Derive the sampling distribution of the sample regression coefficient of any one of the variables on the other, based on a random sample of given size from the $N_2(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, 0)$ distribution. 10+5

6. (a) Let X and Y be iid $N(0, 1)$ random variables. Find the p.d.f. of $\frac{X}{|Y|}$ and also of $\frac{|X|}{|Y|}$.

(b) Let $X_1, X_2, \ldots, X_n$ be iid $N(\mu, \sigma^2)$ random variables. Find the distribution of $Y_n = \dfrac{\sum_{k=1}^{n} kX_k - \mu \sum_{k=1}^{n} k}{\left(\sum_{k=1}^{n} k^2\right)^{1/2}}$.

(c) Derive the distribution of the sample range of a random sample of given size n from the following p.d.f. $f(x) = \begin{cases} e^{-x}; & x > 0 \\ 0; & \text{otherwise} \end{cases}$. 6+4+5

Section - II

Answer ***any two*** from ***question nos.*** **7-10** and ***one*** from ***question nos.*** **11** and **12**.

7. Suppose X_i's ($i = 1, 2, \ldots, n$) are independent with common mean μ and respective variance σ_i^2. Find the BLUE of μ and its variance. 5

8. State Cramer–Rao inequality with regularity conditions. State the condition for equality. 5

9. Write a short note on minimum Chi-square method of estimation. 5

10. Let $X_1, X_2, \ldots, X_n$ be iid random variables with $E(X_i) = \mu$ and $E|X_i|^2 < \infty$. Show that $T(X_1, X_2, \ldots, X_n) = \dfrac{2}{n(n+1)} \sum_{i=1}^{n} iX_i$ is consistent estimator for μ. 5

11. (a) Let $X_1, X_2, \ldots, X_n$ be iid with p.d.f. $f_\theta(X) = \exp[-(x - \theta)]$, $x > \theta$. Consider the class of estimators $T(X_1, X_2, \ldots, X_n) = X_{(1)} + b$, $b \in \mathbb{R}$. Show that the estimator that has smallest MSE in the class is given by $T(\underset{\sim}{X}) = X_{(1)} - \dfrac{1}{n}$.

(b) State and prove factorization theorem for discrete case.

(c) If $X_1, X_2 \underset{\sim}{\text{iid}} P(\lambda)$, show that $X_1 + 2X_2$ is not sufficient for λ. 5+7+3

12. (a) State Rao-Blackwell theorem.

(b) Let $X_1, X_2, \ldots, X_n (n \geq 2)$ be a sample from Bernoulli (p). Find an unbiased estimator for p^2.

(c) Let $X \sim P(\lambda)$. Does there exist an unbiased estimator for $\dfrac{1}{\lambda}$?

(d) Find a sufficient statistic of $\theta \in \Theta = (0, \infty)$ for $U(-\theta, \theta)$ distribution. Justify whether it is consistent or not. 3+3+3+6