# 2022

## PHYSICS

### Paper : PHY 523

### (Astrophysics and Cosmology)

**Full Marks : 50**

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***any five*** questions.

1. (a) The Christoffel symbols can be written as

$$\Gamma^{\sigma}_{\lambda\mu} = \frac{1}{2} g^{\nu\sigma} \left\{ \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} \right\}$$

Find $\Gamma^{t}_{rr}$ and $\Gamma^{\phi}_{\theta\phi}$ for the flat-space FRW metric in spherical polar coordinates

$$ds^2 = -dt^2 + [a(t)]^2 \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right).$$

What is the length dimension of the coordinate $r$?

(b) Let $g_{\mu\nu}$ and $\eta_{\mu\nu}$ be the curved and flat space-time metric respectively, with $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$, and define $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$. Starting from the geodesic equation.

$$\frac{d^2 x^{\lambda}}{d\tau^2} + \Gamma^{\lambda}_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} = 0,$$

where $\tau$ is the proper time, show that in the weak gravity limit, $h_{00} = -2\phi$, where $\phi$ is the Newtonian gravitational potential. You may use the form of $\Gamma^{\lambda}_{\mu\nu}$ as given in 1(a).

(c) The non-zero components of the Ricci tensor for flat FRW geometry in spherical polar coordinates are

$$R_{tt} = -3\frac{\ddot{a}}{a},\ R_{rr} = a\ddot{a} + 2\dot{a}^2,\ R_{\theta\theta} = r^2 R_{rr},\ R_{\phi\phi} = r^2 \sin^2\theta R_{rr}.$$

Find an expression for the scalar curvature $R$.

(d) Starting from the definition of the Einstein tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$, obtain an expression for $R$ in terms of the metric tensor and $G_{\mu\nu}$. The symbols have their usual meanings. (2+1)+3+2+2

2. (a) From the Friedmann equations with a non-zero cosmological constant $\Lambda$, *viz*.

$$\left(\frac{\dot{a}}{a}\right)^2+\frac{k}{a^2}-\frac{\Lambda}{3}=\frac{8\pi G\rho}{3},$$

$$\frac{1}{a^2}\left(2a\ddot{a}+\dot{a}^2+k\right)-\Lambda=-8\pi Gp,$$

Show that a static universe with $\Lambda=0$ must necessarily be empty, but with $\Lambda\neq 0$ one can have a consistent solution.

(b) Show that for $\Lambda\neq 0$, one gets $\frac{\ddot{a}}{a}=-\frac{4\pi G}{3}(\rho+3p)+\frac{\Lambda}{3}$. $\Lambda$ is a constant, but the universe first decelerated between the inflation and approximately 5 billion years, and only after that, started accelerating. Why?

(c) The deceleration parameter $q_0$ is defined as $q_0=-\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2}$. Show that for a matter-filled flat universe with $\Lambda\neq 0, q_0=\frac{3}{2}\Omega_{Mat}-1$.

(symbols have their usual meanings)

(d) Define the critical density of the universe. How does the critical density vary with time for a matter-filled universe? 2+3+3+(1+1)

3. (a) Explain why the energy density of cosmic neutrinos, $E_\nu$, is related to the energy density of cosmic microwave photons, $E_\gamma$, through

$$E_\nu=\frac{21}{8}\left(\frac{4}{11}\right)^{4/3}E_\gamma.$$

(You have to explain both the numerical factors, *i.e.*, 21/8 and $(4/11)^{4/3}$.)

(b) The farthest star that we have observed shows a redshift of 6.2. What was the value of the scale factor $a$ when the light was emitted? What was the temperature and time, then?

(c) State the three Sakharov conditions for baryogenesis. (2+3)+3+2

4. (a) Two black holes, of masses $36M_\odot$ and $29M_\odot$ respectively, merged to form a bigger black hole of mass $62M_\odot$, and the rest of the mass was radiated off as energy contained in gravitational wave. Calculate how much energy was released. If the event happened 400 Mpc away, calculate the energy incident per $m^2$ on earth.

(b) Explain briefly why the uniformity of the CMB temperature is a problem of big bang cosmology.

(c) Derive the Saha equation. Explain why solar spectrum shows more intense Ca II lines than Balmer lines though hydrogen is 500000 more common than calcium in the Sun. (Given Solar temperature is 6000 K and pressure is 15 dynes/cm$^2$. Ca ionization energy is 6.11 eV. Assume $Z_{II}/Z_I=2$ for Ca. For Ca II, K absorption energy is 3.12 eV, and $g_2/g_1=2$) (1+2)+2+(2+3)

5. (a) Deduce the equation of hydrostatic equilibrium for a main sequence star.

Assuming a scaling relation (i.e. assume power law dependence for pressure $P(r)$) and the equation of state for an ideal gas, show that for a Main Sequence star the radius $r \sim M$, the mass of the star. State the assumption involved in obtaining the variation. Also show that in the Main Sequence stage, massive stars have low density and low-mass stars have high density.

(b) Write the equations for the ppI chain. Obtain the abundance ratio of hydrogen and deuterium in equilibrium. (2+4)+(2+2)

6. (a) Obtain an expression for Jeans mass.

(b) Find the Gamow peak energy at a temperature $T$ for the reaction $p + p$.

(c) Using scaling relations, show that the radius of a white dwarf decreases with increase in its mass. Find how the pressure of an ultrarelativistic degenerate electron gas in a stellar core varies with its density. 2+3+(2+3)

7. (a) Explain how we can measure the temperature of a star.

(b) What are the advantages of a reflecting telescope?

(c) A visual binary is observed with a period of nearly 50 years. The ratio of the distances of the stars from the centre of mass of the system is 0.45. The distance of the stars system is 2.9 pc and sustains an angle of 8″. Find the masses of the two stars. Deduce the formula that you use.

(d) Draw the H-R diagram indicating the location of white dwarfs, red giants, O stars and M stars. 2+2+(2+2)+2

You may use :

$G = 6.674 \times 10^{-11}$ N m$^2$ kg$^{-2}$

$k_B = 1.38 \times 10^{-23}$ J K$^{-1}$

$h = 6.626 \times 10^{-34}$ J s

Radiation constant $a = 4\sigma/c = 7.566 \times 10^{-16}$ J m$^{-3}$ K$^{-4}$

Solar mass $M_{\odot} = 2 \times 10^{30}$ kg

$m_H = 1.673 \times 10^{-27}$ kg

1 ly = $9.4607 \times 10^{12}$ km

1 pc = 3.26 ly, 1 Mpc = $3.08 \times 10^{22}$ m.