M. Sc. (Physics) 4th Semester Examination 2019
PHY 523 (Advanced III : Astrophysics and Cosmology)

in your own words as far as practicable. The marks on the right-hand side denote the full marks for the respective question.

arks: 50

Answer **Question no. 1** and any **three** from the rest — Time: 2 Hours

Answer any five: 4 × 5

(a) Show that for the homologous collapse of a cloud of constant density, the free fall time is independent of the mass. Estimate it for the Sun if we assume the Sun to be a homogeneous sphere.

(b) Sirius has the largest apparent brightness of any star in the sky. Assume that the luminosity of Sirius equals that of our Sun. The measured flux of light from Sirius is 1.2×10^{-4} erg/cm^2 sec. Calculate how far away Sirius would be based on our assumption in light years. Predict what the parallax of Sirius would be in this case.

(c) Explain what is meant by Gamow peak. Obtain an expression for the Gamow peak energy.

(d) Suppose the ratio of the gravitational mass and the inertial mass depends on the chemical composition of the object. Schematically describe the Eötvös experiment to show that the torque upon interchanging the weights will no longer be zero if the weights are made of different metals.

(e) (i) If a Schwarzschild black hole is assumed to be a point particle with zero entropy, how does it contradict the second law of thermodynamics?
(ii) In 2017, astronomers observed the merger of two neutron stars in a galaxy 1.4×10^8 light-years away. The light and the gravitational wave (GW) signals came within 2 seconds of each other. Estimate $|1 - (v_{\rm GW}/c)|$, where $v_{\rm GW}$ is the velocity of the GW.

(f) Find the relation between the redshift z and the scale factor $a(t)$. The photon decoupling took place at $z = 1090$. Find the time of decoupling if the universe was matter dominated even at that era.

(a) Obtain an expression for the time taken for a photon created at the centre of the Sun to come out to the surface. Establish the equation for radiative transport.

(b) Show that heat transfer by radiative diffusion implies a non-zero gradient for the reaction pressure proportional to the radiant heat flow. (3+4) + 3

(a) What is the triple alpha reaction? Where does it take place?

(b) Show that the pressure of a degenerate relativistic electron gas is proptorional to $\rho^{4/3}$ where ρ is the matter density.

(c) Using scaling relations, show that the radius of a white dwarf decreases with increase in mass. (2+1) + 4 + 3

4. (a) Establish the equation of motion for a free particle in a curved space-time. From the definition of the Christoffel symbol, show that

$$\Gamma^{\sigma}_{\lambda\mu} = \frac{1}{2} g^{\nu\sigma} \left\{ \frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} \right\},$$

where $g_{\mu\nu}$ is the metric for the curved space-time x^{μ}.

(b) If the metric is given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2\, d\theta^2 + r^2\, \sin^2\theta\, d\phi^2 \right],$$

find $\Gamma^{r}_{\phi\phi}$ and Γ^{t}_{rt}.

(c) The gravitational redshift, with $c = 1$, is given by the difference of the gravitational potentials at emitter and receiver points. Calculate the gravitational redshift if the emitter is on the surface of the sun and the receiver is in vacuum. Use the relevant values given at the end of the paper.

(2+3) + 3 + 2

5. (a) Show that the first Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}$$

and the fluid equation

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

lead to the second Friedmann equation

$$\frac{1}{a^2}\left(2a\ddot{a} + \dot{a}^2\right) = -8\pi G p\,.$$

Take a flat universe with $\Lambda = 0$.

(b) If the equation of state of the cosmological fluid be $p = (\gamma - 1)\rho$ where $0 < \gamma < 2$ is some constant, find how the density and the Hubble parameter change with time. Show that $\gamma = 0$ leads to an exponential expansion. Argue whether $\dot{a} > c$ violate special relativity.

(c) Find the critical density of the universe at the present epoch if $H_0 = 67$ km/s/Mpc.

3 + (2+2+1) + 2

6. (a) Consider the relic abundance problem where the relic is the magnetic monopole. If $\Omega_{mon}/\Omega_{rad} = 10^{-10}$ at $T = 2 \times 10^{29}$ K, when did the matter-radiation transition took place? (Assume all matter in the form of such monopoles; the temperature was still so high that everything else was relativistic.) Since then, the universe was matter-dominated; what is the present value of $\Omega_{mon}/\Omega_{rad}$?

(b) Why does one need the baryon number violating interactions to proceed out of thermal equilibrium for baryogenesis?

(c) Draw the Hertzsprung-Russell diagram for main sequence stars, indicating the different stellar classes.

(d) Explain why the solar spectra show very weak hydrogen Balmer lines although the outer part of the sun is predominantly made of hydrogen.

(2+2) + 1 + 2 + 3

Some numerical values that you may need:

$G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}

$k_B = 1.38 \times 10^{-23}$ J K^{-1} $= 8.6 \times 10^{-5}$ eV/K

$h = 6.626 \times 10^{-34}$ J s

1 Mpc $= 3.26$ light-year $\approx 3.6 \times 10^{11}$ m

Radiation constant $a = 4\sigma/c = 7.566 \times 10^{-16}$ J m^{-3} K^{-4}

$m_H = 1.673 \times 10^{-27}$ kg

Age of the universe $t_0 = 4 \times 10^{17}$ s

CMB temperature at present $= 2.725$ K

Electron mass $m_e = 0.51$ MeV/c^2

Hubble constant $H_0 = 70$ km/s/Mpc

Solar values: $M_\odot = 1.99 \times 10^{30}$ kg, $R_\odot = 6.95 \times 10^8$ m, $L_\odot = 3.84 \times 10^{26}$W,

Solar effective temperature $= 5777$ K.