2022

PHYSICS

Paper : PHY 521

(Nuclear Structure)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer any *five*

1. (a) Express the total antisymmetrized wavefunction of a four fermion non-interacting system in terms of the single particle wave functions. Write the resulting wavefunction when the above total wavefunction is acted upon by a annihilation operator.

(b) For a system of N fermions, with H a symmetric operator, show that,

$$\int \Phi_Q^*(\mathbf{x})\, H\, \Phi_P(\mathbf{x})\, d\tau = \sqrt{N!} \int \Phi_Q^*(\mathbf{x})\, H\, \phi_{p1}(\mathbf{x}_1)\phi_{p2}(\mathbf{x}_2)\ldots\phi_{p5}(\mathbf{x}_5)\ldots\, d\tau\ .$$

$\Phi_Q(\mathbf{x})$ and $\Phi_P(\mathbf{x})$ are wavefunctions of the given system corresponding to distinct sets of single particle states $\{q_1, q_2, \ldots q_5 \ldots\}$ and $\{p_1, p_2, \ldots p_5 \ldots\}$ respectively and $\mathbf{x}$ refers to the set of coordinates of the individual fermions $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$... etc. Note that $\phi_{p1}(\mathbf{x}_1)\,\phi_{p2}(\mathbf{x}_2)$.... denote single particle wave functions. (2+3)+5

2. (a) Explain briefly what you understand by the Hartree-Fock formalism as applicable to a system of weakly interacting fermions.

(b) Derive the Hartree-Fock equations for such a system and find an expression for the total energy in the Hartree-Fock ground state. 2+(5+3)

3. (a) Using Wick's theorem, evaluate

$$\langle 0|B_{ij}D_{kl}F_{mn}c_ic_jc_k^\dagger c_l c_m^\dagger c_n^\dagger|0\rangle.$$

Note that B_{ij}, D_{kl} and F_{mn} are not operators.

(b) The BCS model as applied to nuclei, employs a BCS state expressed as,

$$|BCS\rangle = \prod_{k>0}^{\infty}(u_k + v_k c_k^\dagger\, c_{-k}^\dagger)|0\rangle\ .$$

Each pair of single particle levels $(k, -k)$ is occupied with a probability $|v_k|^2$ and remains empty with a probability $|u_k|^2$. Evaluate, in terms of u_k and v_k,

Please Turn Over

i. the normalization,

ii. the particle number, and

iii. the particle number uncertainty, in reference to the BCS state.

2+(2+2+4)

4. (a) Obtain the surface term in the Fermi gas model.

(b) Assume that the two valence neutrons in ^{18}O are allowed only in the $1d_{5/2}$ and $2s_{1/2}$ single particle states. Find out the spin-parity values of all the possible states of the nucleus. Show that the states with the same spin-parity can never be degenerate in presence of a two body residual interaction.

(c) Find out the allowed J values for three identical particles in the single particle state $j = 5/2$.

3+(2+2)+3

5. (a) Show that in the nuclear collective model, the dipole deformation at the lowest order corresponds to a shift of the centre of the mass but no actual deformation.

(b) The stretching of the Cartesian axes in a nucleus with quadrupole deformation is given by

$$\Delta R_k = \sqrt{\frac{5}{4\pi}}\beta\cos(\gamma - 2\pi k/3)$$

where β and γ are the usual quadrupole deformation parameters. Show that γ values lying between 0 and $\pi/3$ are sufficient to describe all possible shapes.

(c) Show that for surface quadrupole vibration,

$$\langle r^2\rangle = \frac{3}{5}R_0^2 + \frac{3}{4\pi}R_0^2\langle\beta^2\rangle$$

where R_0 is the radius and $\langle\beta^2\rangle$ is the mean square deformation.

3+3+4

6. (a) In the transverse gauge, the electric and magnetic multipole fields may be written as

$$\vec{A}_{lm}(\vec{r}, E) = \sqrt{\frac{l+1}{2l+1}}j_{l-1}(kr)Y_{lm,l-1}(\Omega) - \sqrt{\frac{l}{2l+1}}j_{l+1}(kr)Y_{lm,l+1}(\Omega)$$

$$\vec{A}_{lm}(\vec{r}, M) = j_l(kr)Y_{lm,l}(\Omega)$$

in terms of Bessel functions and vector spherical harmonics.

i. Show for the electric and magnetic intensities for the electric transitions,

$$\vec{E}(\vec{r}, E) = ik\vec{A}(\vec{r}, E)$$
$$\vec{H}(\vec{r}, E) = -ik\vec{A}(\vec{r}, M)$$

ii. Explain why, for low energy gamma transitions, we may write

$$\vec{A}_{lm}(\vec{r}, E) \approx \sqrt{\frac{l+1}{2l+1}} j_{l-1}(kr) Y_{lm,l-1}(\Omega)$$

(b) Find out the most probable transitions (electric/magnetic and multipolarity) for the following transitions:
i. $2^+ \longrightarrow 1^-$ ii. $5/2^+ \longrightarrow 3/2^+$ (4+2)+(2+2)

7. (a) Deduce an expression for the effective charge of protons and neutrons in gamma decay.

(b) The Lagrangian density for MIT Bag model is given by

$$\mathcal{L} = [\frac{i}{2}(\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - (\partial_{\mu}\bar{\psi})\gamma^{\mu}\psi) - B]\theta_v(x) - \frac{1}{2}\bar{\psi}\psi\Delta_s$$

where $\theta_v = \theta(R - r)$ is a step function and $\Delta_s = \delta(R - r)$ is a delta function. Here, Ψ is the quark wave function, R is the radius of the bag and B is the bag constant.
Show that the quark probability density vanishes at the bag surface.

(c) Find out the energy eigenvalues for an axially symmetric rigid rotor.

4+3+3