2016

STATISTICS — GENERAL

Second Paper

Full Marks – 100

The figures in the margin indicate full marks

Candidates are required to give their answers in their own words as far as practicable

SET – II

Group – A

Answer ***Question No. 1*** and ***any three*** questions from the rest

1. Answer ***any four*** of the following questions : 2×4

(a) What do you mean by 'random sampling' ?

(b) What is standard error of an estimator ?

(c) Define 'power' of a test.

(d) Define efficiency of an estimator.

(e) Write down the p.d.f. of a t distribution with 10 degrees of freedom.

(f) In which situation, you can use large sample tests ?

(g) If X_1 follows Poisson (m_1) and X_2 follows Poisson (m_2) independently, then write down the distribution of $X_1 + X_2$.

(h) What is Fisher's t-test ?

2. (a) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a normal distribution with mean μ and standard distribution σ. How do you perform the following tests ?

(i) $H_0 : \mu = 5$ against $H_1 : \mu > 5$

(ii) $H_0 : \mu = 5$ against $H_1 : \mu \neq 0$

(iii) $H_0 : \sigma = 1$ against $H_1 : \sigma > 1$.

(b) Given two variables, how do you test whether there is no correlation between the variables based on a random sample. (It is assumed that two variables jointly follow bivariate normal distribution). (3+3+3)+5

3. (a) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with p.d.f. $f(x)=\frac{1}{\theta}e^{-x/\theta}$, $0<x<\infty$.

Obtain the maximum likelihood estimator of θ.

(b) What is the moment estimator of θ ?

(c) Check whether the estimator obtained in (a) and (b) one unbiased or not. 5+5+4

4. Suppose X_1 and X_2 are independently distributed normal variables with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2.

(a) How do you test

(i) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$, when σ_1 and σ_2 are known

(ii) $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$, when σ_1 and σ_2 are unknown

(iii) $H_0 : \sigma_1 = \sigma_2$ against $H_1 : \sigma_1 \neq \sigma_2$, when μ_1 and μ_2 are unknown.

(b) Obtain confidence interval for $\mu_1 - \mu_2$ and $\frac{\sigma_1}{\sigma_2}$. (3+3+3)+5

5. (a) Suppose two independent random variables X_1 and X_2 are such that X_1 follows $N(\mu_1, \sigma^2)$ and X_2 follows $N(\mu_2, \sigma^2)$. Obtain the distribution of

(i) $ax_1 + bx_2 + c$

(ii) $\frac{1}{\sigma^2}\left[(X_1-\mu_1)^2 + (X_2-\mu_2)^2\right]$.

(b) If $X_1, X_2, \ldots, X_n$ are independent $\text{Bin}(n, p)$ variables, then what is the distribution of $\sum_{i=1}^{m} X_i$?

(c) What is the maximum likelihood estimator of σ^2 based on a random sample of size n drawn from $N(0, \sigma^2)$? Is this estimator unbiased ? (3+3)+3+5

6. (a) How do you use Pearsonian χ^2–statistic to test

(i) the independence of two categorical variables

(ii) the homogeneity of several similarly classified populations.

(b) What is p-value ? How do you use this value in testing of hypothesis ? (5+5)+4

7. Write notes on ***any two*** of the following : 7×2

(a) χ^2–distribution

(b) Chi-square test for goodness of fit

(c) Interval estimation.

Group – B

Answer ***Question No. 8*** and ***any three*** questions from the rest

8. Answer ***any four*** of the following questions : 2×4

(a) What is OC curve ?

(b) Write down Fisher's quantity index number.

(c) Give two criteria for an ideal base period.

(d) Write down the normal equations for fitting a straight line to a time series data using least square method.

(e) Define q_x column of a life table.

(f) What is wholesale price index number ?

(g) "The cost of living index number for the industrial labour during 2012–13 with 2001–02 as the base is 201.3". — Explain.

(h) Give the multiplicative model of a time series with respect to different systematic components of the time series.

9. (a) Discuss about the different sources of vital events.

(b) Distinguish between the terms 'rate' and 'ratio' with respect to different vital events.

(c) Show that, $L_x = \frac{l_x + l_{x+1}}{2}$ under appropriate assumption, where l_x and L_x are two usual columns of a life table. 6+4+4

10. (a) Discuss different components of a time series.

(b) How do you fit a pth degree polynomial to a time series data using least square method ? 8+6

[Turn Over]

11. (a) What are 'Time reversal test' and 'Factor reversal test' ?

(b) Give three index number formula in which one satisfies both the tests in (a) but the remaining two satisfy any one of the tests.

(c) What is 'cost of living' index number ? (3+3)+6+2

12. (a) What do you mean by 'control chart' in statistical quality control ?

(b) Discuss the control charts for

(i) Number of defects

(ii) Number of fraction defectives.

for both the cases standard given and not given. 2+(3+3+3+3)

13. (a) Discuss 'ratio to moving average' method for determining seasonal indices for quarterly data.

(b) Distinguish between 'Gross reproduction rate' and 'Net reproduction rate'.

(c) Define AOQL for single sampling inspection plan. 5+6+3

14. Write short notes on ***any two*** of the following : 7×2

(a) Standardised death rate

(b) Problems in construction of index numbers

(c) Single sampling inspection plan.