S(1st Sm.)-Physics-411/(CBCS)

14/1/19

# 2018

## PHYSICS

### Paper : PHY-411

### (Mathematical Methods)

### Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***any five*** questions.

1. (a) Find the rank and nullity of the matrix $A=\begin{pmatrix}1 & 2 & 0 & 1 & 1\\ 2 & 4 & 1 & 3 & 0\\ 4 & 8 & 1 & 5 & 2\end{pmatrix}$.

   (b) Consider a discrete finite group $G$ under a group composition law denoted by o. Prove that in any row or column of the group composition table, every distinct element of $G$ may appear only once.

   (c) The Laplace transform of a function $f(x)$ is defined as,

   $$\mathcal{L}\left\{f(x)\right\}=\int_0^\infty dx e^{-sx} f(x).$$

   Find $\mathcal{L}\left\{\int_0^x g(t)dt\right\}$, clearly stating all assumptions made. 4+3+3

2. (a) Consider the vector space of 2×2 matrices $M_{2\times2}$. Does the set $K$ of all matrices of the form $\begin{pmatrix}2a & b\\ 3a+b & 3b\end{pmatrix}$, with $a$ and $b$ real, form a subspace of $M_{2\times2}$?

   (b) Verify if the vectors $p_1(x)=x+1$, $p_2(x)=x-2$ and $p_3(x)=x^2-1$ are linearly independent.

   (c) What are the eigenvalues of the matrix $A=\begin{pmatrix}3 & -1 & -2\\ 2 & 0 & -2\\ 2 & -1 & -1\end{pmatrix}$?

   If the matrix is diagonalizable, find the diagonalizing matrix $P$ such that $P^{-1}AP$ is a diagonal matrix. [3+2+(2+3)]

3. (a) Consider the group of permutations of three objects, $S_3$. Identify all the subgroups and their order. For a nontrivial proper subgroup of your choice, write down a faithful matrix representation.

   (b) Generate a group from two elements $A$ and $B$ subject only to the relation $A^2=B^3=(AB)^2=E$, where $E$ is the identity of the group.

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(c) Show that a rotational transformation of vectors in a plane through an arbitrary angle $\theta$ may be expressed as $\exp(i\,\theta\,\sigma_2)$, where $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$. (2+2)+3+3

4. (a) Let $G = \{g_1, g_2, \ldots, g_k\}$ be a discrete finite group under a group operation denoted by o, where each element is its own inverse. Prove that the group is Abelian, i.e., $g_r \circ g_s = g_s \circ g_r$. Use of $(g_r \circ g_s)^{-1} = g_s^{-1} \circ g_r^{-1}$ is highly discouraged.

(b) The Fourier transform of a function $f(t)$, denoted by $F(\omega)$, may be defined as,

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, e^{i\omega t} f(t).$$

Find the Fourier transform of $f'(t)$, where $'$ implies derivative w.r.t. $t$. State clearly all assumptions made.

(c) The equation $y''(x) + P(x)\,y'(x) + Q(x)\,y(x) = 0$ has a solution $y_1(x)$. Assume a second solution of the form $y_2(x) = y_1(x)\,f(x)$ and substitute this into the original equation to obtain an expression for $f(x)$. Hence, given the solution $y_1(x) = 1$ for the Legendre equation $(1-x^2)y'' - 2xy' + n(n+1)\,y = 0$ for $n = 0$, obtain the second solution. 3+2+(3+2)

5. (a) Find the Green function for the equation $d^2y(x)/dx^2 + f(x) = 0$, subject to the boundary conditions $y(0) = 0$, $y(1) = 0$. Hence solve the equation $d^2y(x)/dx^2 + x^2 = 0$ with these boundary conditions.

(b) Transform the equation $y'' - \frac{1}{x^2}y' + \frac{2}{x^3}y = 0$ into self-adjoint form.

(c) Show that sinh$x$ and cosh$x$ are linearly independent. (3+3)+3+1

6. (a) Evaluate $\int_0^{\infty} \frac{\cos(ax)\,dx}{x^2 + b^2}$ where $a, b > 0$, by the method of contour integration.

(b) Find the residue of the function $1/(z^2+1)^2$ at $z = i$. Also determine whether this function has a singularity at the point at infinity.

(c) Find the Laurent series expansion of the function $\frac{1}{z(z-1)}$ about $z = 0$ for $|z| > 1$. 5+(2+1)+2

7. (a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$ by the method of contour integration.

(b) The imaginary part of an analytic function $f(z)$ is $4(x^3y - xy^3)$. Find the real part. Express $f(z)$ as function of $z$.

(c) Expand $(1/z)\sin(1/z)$ about $z = 0$ and identify the type of singularity. 5+(3+1)+1