2022

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PHYSICS

Module : PHY-421

(Classical Electrodynamics)

Full Marks : 50

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Answer ***any five*** questions.

1. (a) Two concentric rings with radii 'a' and 'b' respectively, (a>b) are placed on xy-plane keeping their centers at the origin. Charges of +Q and –Q are distributed uniformly over the inner and outer rings respectively. Construct the quadrupole moment tensor of this charge distribution. Will it depend on the choice of origin?

   (b) Calculate the surface current density that produces the potentials $\Phi = 0$ and $\overline{A} = \frac{\mu_0 \alpha}{4c}(ct - |z|)^2\,\hat{x}$; for $ct > |z|$ and zero otherwise.

   (c) A 1 kW monochromatic point source radiates uniformly in all directions in space and is detected at a distance of 5 meter from the source. What is the amplitude of the electric field?

   (d) Starting from the electromagnetic force on the charges and current, derive the expression for Maxwell stress tensor in the absence of electrostriction and magnetostriction. 3+2+2+3

2. (a) Starting from the expressions of retarded potentials as the solution of inhomogeneous equations, derive the expression of electric field. Justify why the radiation fields vary as inverse of distance from source to the point of detection.

   (b) Vector potential of center fed antenna is given by : $\overline{A}(\overline{R}) = \hat{z}\,\frac{\mu_0 I}{2\pi}\,\frac{e^{ikR}}{kR}\,\frac{\cos\left[(kd/2)\cos\theta\right] - \cos(kd/2)}{\sin^2\theta}$, where symbols have their usual meaning. Determine the radiation fields. Hence calculate the time averaged angular power distribution for full wave antenna. If the time averaged total power is $0.83\frac{\mu_0 c I^2}{\pi}$, determine the directivity of this antenna.

   (c) Show that for an arbitrary source, scalar potential at a far distance can be approximated as : $\Phi(\overline{R}, t) = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q(t_0)}{R} + \frac{1}{Rc}\dot{\overline{p}}(\overline{r'}, t_0).\hat{n} + \frac{1}{R^2}\overline{p}(\overline{r'}, t_0).\hat{n}\right]$, where Q and $\overline{p}$ represent monopole and dipole moment of the charge distribution. 3+4+3

3. (a) Starting from the relevant magneto-hydrodynamic equations, derive the equation :

$$\frac{\partial \overline{B}}{\partial t} = \overline{\nabla} \times \left(\bar{v} \times \overline{B}\right) + \eta_m \nabla^2 \overline{B}.$$ Justify why n_m is called magnetic viscosity.

(b) Indicate how the sausage instability can be overcome by a longitudinal magnetic field.

(c) Calculate the Debye length in a plasma system taking account of different electron and ion temperatures.

(d) Derive the expression of relative permittivity in a plasma medium. 3+2+3+2

4. (a) How do electric field and magnetic field transform under parity, time reversal and charge conjugation?

(b) An exponentially decaying point charge $\left(q(t) = q_0 e^{-\frac{t}{T}}\right)$ is at rest at the point $(b, 0)$. What is the scalar potential ϕ and vector potential $\overrightarrow{A}$ as felt by an observer at $(0, b)$ who is moving with a velocity $\vec{v} = -\hat{y}\, v$, with respect to the charge.

(c) Calculate the minimum energy required for a proton (rest mass approximately equals to 1 GeV) to emit Cerenkov radiation when it passes through glass (R.I. 1.5). 4+3+3

5. (a) Electrically neutral yz plane carries a time dependent but uniform current $J(t)\hat{z}$, switched on at $t = 0$. Show that retarded vector potential at a height x above the plane is

$$A(x, t) = \frac{1}{2}\hat{z}\int_0^{ct - x} J(\xi)\, d\xi.$$

Calculate the power radiated by unit area of the surface at a time $t > 0$.

(b) Determine the fields $\overrightarrow{E}\left(\vec{r}, t\right)$ and $\overrightarrow{B}\left(\vec{r}, t\right)$ due to the electric dipole moment of the time varying source. Determine the angular distribution of the power radiate and the total power radiated. (3+3)+(2+2)

6. (a) In an inertial frame, components of electric and magnetic fields due to a charge and current distributions are (2,3,5) and (0,0,0) respectively, in some chosen system of unit. In another inertial frame, which is moving with uniform speed with respect to the former, components of magnetic field are (2,1,3). What may be the components of the electric field in the second frame of reference?

(b) Two large parallel plates (parallel to x-axis) separated by a distance d, have uniform surface charge density $+\sigma$ and $-\sigma$ respectively in the rest frame of the plates. What are the electric and magnetic fields between the plates as seen by an observer moving at a uniform speed of v along the x-axis w.r.t. plates? Neglect any effects due to finite size of the plates. Derive the necessary formula for transformation of electric and magnetic fields. 4+(3+3)

7. (a) Electric field $\vec{E}$, due to a non-relativistic accelerated charge is, $\vec{E} = \left(\frac{e}{4\pi\epsilon_0 R^3}\right)\vec{R} \times \vec{R} \times \vec{a}$. Here, $\vec{R}$ is the vector joining the position of the charge and the observer while $\vec{a}$ is the acceleration of the charge at an instant of time.

Use this formula to calculate the frequency spectrum of emitted radiation when a uniformly moving charge, e, is being scattered by a heavy target at $t = 0$ such that its speed is being changed as $v = v_0\, e^{-t/T}$ for $t > 0$.

(b) Lagrangian (density) for a complex scalar field is, $\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 |\phi|^2$. Find the Euler-Lagrange equation for ϕ^*. Calculate the momenta conjugate to field variables ϕ and ϕ^*. Calculate Hamiltonian (density) for the above Lagrangian. 5+(2+2+1)