# 2022

## PHYSICS

### Module : PHY-422

### (Quantum Mechanics II)

### Full Marks : 50

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

Answer ***any five*** questions.

1. (a) A particle of mass $m$ is initially in the ground state of one dimensional box with infinitely high walls at $x = 0$ and $x = a$. The system is subjected to a time dependent perturbation $V(t) = x^2 e^{-\frac{t}{\tau}}$ ($\tau$ is a constant). Calculate, up to the first order, probability of finding the particle in the first excited state at a later time $t$ ( $t >> \tau$).

(b) Given the expression for transition probability is $P_{if}(t) = \left|\left(\frac{i}{\hbar}\right)\int_0^t \langle f|V(t')|i\rangle e^{i\omega_{fi}t'} dt'\right|^2$ where $f$ and $i$ are final and initital states respectively and $\omega_{fi} = (E_f ? E_i) / \hbar$, $E_f$ and $E_i$ are final and initial energies.

Applying harmonic perturbation, find the expression for the transition probability consisting of resonance and anti-resonance terms. Hence interpret the origin of stimulated emission and stimulated absorption from the derived expression. 6+4

2. (a) Two identical particles of spin-1/2 are moving under the influence of a one dimensional harmonic oscillator potential. Assuming that the two particle system is in the triplet spin state, find the energy levels, the wave functions and the degeneracies corresponding to three lowest states.

(b) Show that the wave function for non-degenerate eigenstates are always real for a Hamiltonian which is invariant under time reversal. Explain why a free particle wave function can be described by $e^{ipx}$, in spite of the fact that a free particle Hamiltonian is invariant under time reversal. 6+4

3. (a) (i) Show for a 4-vector $A$, $\not{A}\not{A} = A^2$. Here, $\not{A} = A_\alpha \gamma^\alpha$.

(ii) Show that $\Lambda_+ = \frac{1}{2mc}(\not{p} + mc)$ and $\Lambda_- = \frac{1}{2mc}(-\not{p} + mc)$ are a complete set projection operators and they project over positive and negative energy solutions of Dirac equation.

(b) Calculate the conserved flux 4-vector that follows from free Klein Gordan equation. Argue why cannot the time component of this 4-vector be identified with the probability density of the finding the particle in space? [2+(2+2)]+(3+1)

4. (a) In non-relativistic limit, show that the Dirac equation for an electron (with charge $-e$) in a magnetic field $\vec{B}$ reduces to

$$\left(\frac{1}{2m}\left(\vec{p}+\frac{e}{c}\vec{A}\right)^2+\frac{e}{2mc}\vec{\sigma}.\vec{B}+mc^2\right)\psi=E\psi.$$

Here, $\vec{A}$ is the magnetic vector potential and $\vec{B}=\vec{\nabla}\times\vec{A}$. $\vec{p}$ and $m$ are the 3-momentum and rest mass of the electron respectively.

(b) A proton of energy $E$ is incident from the right on a nucleus of charge $Ze$. Using the WKB approximation, estimate the transmission coefficient associated with the penetration of the proton inside the nucleus. 5+5

5. (a) Using the Green's function, obtain the total scattered wave function in the centre of mass frame.

(b) What is the Born series? Interpret the Born series with the diagram.

(c) Consider the scattering of a particle of mass $m$ from a hard sphere potential :

$V(r)=\infty$ for $r\leq a$ and $V(r)=0$ for $r>a$.

Determine whether the potential is of attractive or of repulsive nature. Show that the total scattering cross section in the low energy limit is four times the classical value. 4+3+3

6. (a) For a spherically symmetric scattering potential, using the partial wave analysis, establish that the total elastic scattering cross section is given by

$$\sigma_{el}=\frac{4\pi}{k^2}\sum_l(2l+1)\sin^2\delta_l$$

(b) Find the differential and total cross section for the scattering of slow (small velocity) particles from a spherical delta potential $V(r)=V_0\,\delta(r-a)$ (use partial wave analysis). Discuss what will happen if there is no scattering potential.

(c) Show that for $j=1$, it is legitimate to replace $e^{-\frac{i}{\hbar}\hat{J}_y\beta}$ by 3+4+3

$$\mathbb{1}-\frac{i}{\hbar}\hat{J}_y\sin\beta-\left(\frac{\hat{J}_y}{\hbar}\right)^2(1-\cos\beta).$$

7. (a) Why do we use spherical tensors for studying transformations under rotations instead of Cartesian tensors?

Use Cartesian to Spherical transformation to show that

$$r_{+1}^{(1)} = r\sqrt{\frac{4\pi}{3}}\, Y_{1,+1}(\theta, \phi)$$

$$r_{0}^{(1)} = r\sqrt{\frac{4\pi}{3}}\, Y_{1,0}(\theta, \phi)$$

$$r_{-1}^{(1)} = r\sqrt{\frac{4\pi}{3}}\, Y_{1,-1}(\theta, \phi)\,.$$

[Given that $Y_{0,0} = \sqrt{\frac{1}{4\pi}}$, $Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta$, $Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\exp^{\pm i\phi}$]

Hence find the dipole operator.

(b) Find the most general unitary uni-modular matrix in terms of Cayley Klein parameters for the group $SU(2)$. Hence show that the matrix can be interpreted as representing a rotation (given $j = \frac{1}{2}$).

(c) A particle in a central potential has an orbital angular momentum $l = 2\hbar$ and a spin $s = 1\hbar$. Find the energy levels and degeneracies associated with a spin-orbit interaction term of the form $H_{so}$ = AL.S, where $A$ is a constant. (1+2+2)+3+2